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Report No. 876
Job No. 110723

ON MEASURING TRANSDUCER CHARACTERISTICS IN A WATER TANK

P. W. Smith, Jr.
T. J. Schultz

Contract Nonr 3468(00)
Task NR 385-424

8 September 1961

Submitted to:

Head, Acoustics Branch
Earth Sciences Division
Office of Naval Research
Code 411
Washington 25, D. C.

Attention: Mr. Marvin Lasky

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ON MEASURING TRANSDUCER CHARACTERISTICS IN A WATER TANK

ABSTRACT

The concepts and procedures of room acoustics are reviewed and adapted to underwater sound problems. These results are used in an attempt to answer engineering questions about the design of calibration tanks.

Presently available procedures for predicting the far-field characteristics of a transducer from near-field measurements are described and critically reviewed.

Studies are made of: (1) sound absorption by mechanically damped plates; (2) the equations for energy functions in a steady sound field; and (3) the capabilities and applications of intensity meters.

An outline is given of areas requiring further study before substantial progress is possible.

Unfortunately, because of fundamental ignorance about certain aspects of tank acoustics and the near fields of transducers, most of the quantitative results are quite uncertain. This accounts for the urgency for further detailed study.

ON MEASURING TRANSDUCER CHARACTERISTICS IN A WATER TANK

I. INTRODUCTION

A. GENERAL

This report is a broad and preliminary investigation of the inherent limitations on attempts to determine the free-field characteristics of underwater sound transducers from steady-state measurements made in a tank, i.e., any body of water of quite limited extent. The question has been precipitated by the design and construction of low-frequency (5 kc/s and below) sonar transducers, and by the natural desire to use existing facilities for their test. The problem has been very well outlined in a series of papers by Klein (1959, 1960a, 1960b).*

In its general aspects, such an investigation constitutes an adaptation to water-filled tanks of the considerations of "room acoustics", developed for air-filled volumes. (It is interesting to note that the problem of measurement in small, non-ideal volumes of air, has been one of increasing practical interest over the past five years.) This report begins in Chapter II, with a survey and critical review of the fundamental concepts and theoretical predictions of room acoustics, adapting them to the water tank problem.

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*Citations are made by author's name and year of publication, with a letter suffix to distinguish chronologically successive publications in the same year. The list of references is arranged alphabetically by author and chronologically for each author.

In Chapter III these results are used in an attempt to answer some engineering questions pertinent to tank design. Where answers are not possible, for lack of basic knowledge, guesses are made. From these considerations, numerical limits are computed for the tank size required for a typical directive transducer (Section 5, Summary).

Chapter IV is a collation and comparison of the various schemes that have been proposed for measuring the characteristics of transducers. These methods include measurements in reverberant tanks and measurements in the transducer's near field, in an environment where the reverberant energy is small. Inherent limitations of each method are discussed.

The acoustical characteristics of a tank are, of course, fundamentally dependent on the sound absorption of its walls. Chapter V summarizes available information on the low-frequency absorption, both that naturally inherent in the tank's construction and that due to various applied treatments. In the latter connection, a preliminary study is made of the absorptive possibilities afforded by mechanically damped plate structures (Chapter VI).

It has been recurrently necessary throughout this survey to confess our ignorance of important fundamental characteristics of the acoustics of transducers and tanks. These topics requiring further basic inquiry are collated in Chapter VII. Two of them have been considered in some detail in appendices. Appendix I investigates the equations governing energy functions of a steady sound field. Appendix II is a broad survey of the history, capabilities, and possible applications to underwater sound measurements of intensity measuring devices.

B. CALIBRATION REQUIREMENTS

This report is, by direction, focused on the problems of making calibration measurements on large, low-frequency transducers. However, as the difficulties in making such measurements in tanks of reasonable size are developed, a different question insistently recurs: Are such measurements necessary?

An unequivocal answer cannot be made; the measurements required depend upon the purpose of the test. As Trott (1960) pointed out in an excellent discussion of the question, there are four principal purposes for sonar calibration tests:

- research
- prototype development
- production
- maintenance and repair

There is no a priori reason why the same tests should be required for each purpose.

Trott has suggested that the tests required for research and development may be much more elaborate than those required for either production or maintenance. Such a distinction is one of very great potential importance. As transducers grow larger, it is inevitable that the existing tanks be abandoned as inadequate for testing complete transducers. A search should be instituted now for new techniques of production and maintenance test that do not require new, larger tanks but will still maintain adequate quality control. Incidentally, such a distinction between tests for R and D and tests in production is common in all technological industries.

II. SOUND FIELDS IN TANKS

In this chapter we review critically the fundamental concepts and theoretical predictions of room acoustics, with particular attention to adapting them to water-filled tanks.

A. FUNDAMENTAL CONCEPTS

The question with which we are concerned is the determination of the radiation properties of a transducer from measurements of its sound field in a tank. This is the inverse of the classical problem of room acoustics wherein the source is known and the sound field in the room is to be determined. Both the knowledge and, unfortunately, the uncertainties and ignorance which exist in room acoustics are therefore generally pertinent to our question. In this section we shall discuss briefly some of the fundamental concepts used to describe sound waves in rooms.

1. Field of Transducer in Free Space

First consider a finite transducer in an infinite medium without boundaries. The resulting sound field can be divided into several regions whose boundaries are not sharp.

a) The far field:

"Far enough" from the transducer, the sound pressure field can be accurately described as the product of (1) a directivity function dependent only on angle, (2) r^{-1}, where r is range, and (3) a constant.

The range and angle are measured from the "acoustic center," a point near the transducers, to the observation point.* This region "far enough" away from the source is called the far field.

The restrictions on range r for a point to be in the far field have been stated to be twofold (Stenzel, 1958; Part 3).

$$r^2 \gg a^2 \text{ or,}$$

$$\kappa_1^2 \gg 1, \text{ with } \kappa_1 \equiv \frac{r}{a} = \frac{r/\lambda}{a/\lambda} = \frac{kr}{ka} \quad ; \text{ and} \qquad (1)$$

$$\lambda r \gg \pi a^2 \text{ or,}$$

$$\kappa_2^2 \gg 1, \text{ with } \kappa_2^2 \equiv \frac{\lambda r}{\pi a^2} = \frac{r/\lambda}{\pi(a/\lambda)^2} = \frac{2kr}{k^2a^2} \quad ; \qquad (2)$$

where 2a is the largest dimension of the source, r is range from acoustic center, λ is wavelength of sound, and $k=2\pi/\lambda$.

---

*However one must find the correct location of the acoustic center or the characteristics of the far field will not obtain. The Lloyd mirror effect, and its analog in the field radiated from a dipole source, is a good example of the errors incident to a misidentification of the acoustic center. The source is a dipole and if one measures pressure at points near the null plane in the radiation pattern the following anomaly appears. Along a radius vector from the true acoustic center (midway between the two halves of the dipole) the pressure varies as r^{-1}. Along some radii from false centers, pressure may vary as r^{-2}. Such peculiar sensitivity to accurate identification of the acoustic center is not to be expected near maxima in the radiation pattern.

In actuality, for reasons discussed in part (c) below, these restrictions may not be sufficient. At least in the case of small multipole sources one must also add

$$k^2r^2 \gg 1 \quad . \tag{3}$$

In addition to those characteristics of the far sound field listed above, one finds in the far field that (1) the phase angle between pressure and particle velocity tends to vanish; (2) the wave specific acoustic impedance in a radial direction tends to (ρc); and (3) the kinetic energy density is equal to the potential energy density so that the Lagrangian density (kinetic less potential) vanishes.

b) <u>The near field; Fresnel zone</u>:

When the source is large, or even moderately large, in terms of wavelengths:

$$ka \geqslant 2\pi \quad , \tag{4}$$

there are regions in which Eqs. (1) and (3) are reasonably well satisfied but Eq. (2) is not.

In <u>most</u> cases, such a region is characterized by large fluctuations of sound pressure with position. These arise from phase cancellation and reinforcement between the contributions to pressure from the various parts of the source. The fluctuations are not describable in a <u>simple</u> manner in terms of a single spherically diverging wave. Associated with the fluctuations is a large reactive component of the sound field; that is, pressure and velocity are not in phase and the time-averaged Lagrangian density does not vanish. This behavior is probably typical of the sound field of any large directive sound source.

However in the case of a non-directive source (a uniformly excited sphere), Fresnel diffraction effects do not arise, no matter how large the source radius a. Thus it appears that the restriction expressed in Eq. (2) may be relaxed in some cases while the simple far field prediction methods are retained. The extent of the relaxation is undoubtedly intimately related to the directivity of the source. The problem per se has not been studied in any detail although considerable information is available about some particular source geometries (Stenzel).

c) The reactive near field without interference:

When the source is small in terms of wavelengths a different "near field" effect is found. We consider here regions in which Eq. (3) is not satisfied

$$kr \leq 1 \quad , \tag{5}$$

while Eq. (2) is satisfied. Eq. (1), i.e., r/a large, may or may not be satisfied in such regions.

The typical example of this situation occurs with a small source which may be a monopole (simple source), dipole, or of higher order. Consider first the monopole source. It is a somewhat special case in that the sound pressure varies as r^{-1} at all distances, even where kr is small or where $r \doteq a$. However when kr is small the particle velocity varies faster (in space) than r^{-1} and is not in phase with the pressure. The Lagrangian density does not vanish, there being an excess of kinetic energy.

Next consider small sources of higher order, dipole, quadripole, etc., i.e., directive sources. The r^{-1} law of variation of pressure is no longer true when kr is small (Morse and Feshbach, 1953, p. 1575). However in a pure n-pole (i.e., of single order) the angular distribution of pressure is the same at all distances r; the variation of pressure with r is the same in each direction although different for sources of different order. One concludes, therefore, that in the case of a general small source, including component n-poles of several orders, the angular distribution of pressure will vary with r if, and only if, kr is small.

The quantitative importance of such a variation with r of the angular distribution of pressure in the case of practical small sources is a matter distinct from the existence of the variation. Qualitatively, one expects a small source to radiate most effectively as a monopole and ineffectively as an n-pole of higher order.

2. Field of Transducer in a Tank

In the previous section we considered the sound field generated by a specific source in an infinite space. Now let us consider the same source installed in a tank.

a) Direct and reverberant fields

A fundamental concept in room acoustics is the decomposition of the total sound field, the potential ψ_t, into direct ψ_d and reverberant ψ_r components:

$$\psi_t = \psi_d + \psi_r \quad . \tag{6}$$

The direct field is defined as identical to the field generated in an infinite space by the same excitation so that the previous discussion is pertinent to it. The reverberant field is then uniquely determined (in principle) by reciprocity, if the response of the medium and tank walls is linear and the characteristics of each are specified. The reverberant field is the sound field that would build up in the tank, with the source off, in response to a carefully selected distribution of sources on the tank walls. The selection must be made in a manner to ensure that the total field, Eq. (6), satisfies the actual boundary conditions at the tank wall. (Fig. 1)

Such a procedure is not recommended for analysis (except in the high frequency limit), but it may be useful in qualitative considerations. For example, in a tank with concave walls, one might well suspect that the reverberant field will be partly concentrated or focussed in certain regions. If the source is located at a focus, the reaction of the reverberant field on the source may have adverse effects. On the other hand, if one hopes to make measurements in a region where the direct field predominates, it is probably advantageous, in a tank of fixed volume, to have foci at points not near the source.

The distinction between "direct" and "reverberant" fields which we have made here is fundamentally the same as that made by Beranek in his analysis of reverberant sound energy density (Beranek, 1954, section 10, 13). However, Beranek proceeds upon an analysis whose validity may be challenged except as an approximation for high frequencies and a sufficiently irregular room. It appears to be based on the assumption that the reverberant field by itself is uniformly distributed in a random (ergodic) fashion.

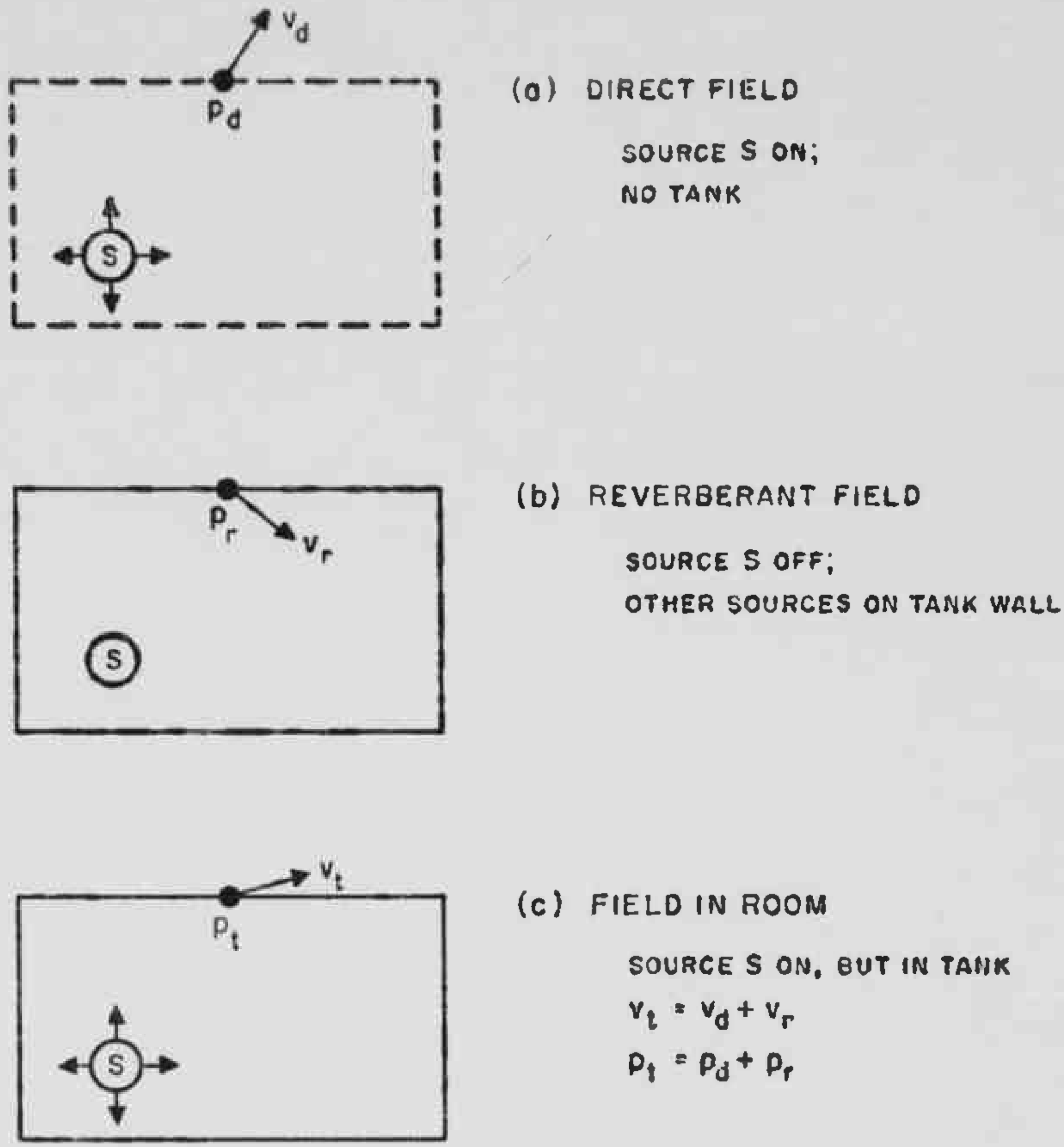


FIG. 1 SCHEMATIC REPRESENTATION OF DECOMPOSITION OF SOUND FIELD IN TANK INTO DIRECT AND REVERBERANT FIELDS

In the literature on architectural acoustics it is commonly assumed, at least implicitly, that the reverberant field, as defined in this section, is a standing wave field which does not carry real power; by corollary, therefore, the real power flow is described by the direct field. Sometimes this assumption is considered a valid first approximation. [See, inter alia, Morse and Bolt, 1944, section 33; Bolt and Roop, 1950; Pachner, 1956b; Doak, 1959.] Unfortunately this assumption appears not to be valid even as a first approximation.

Near a source, where the direct field predominates over the reverberant field, the power flow will, indeed, be described by the properties of the direct field. However in the simple case of a room whose walls are uniformly covered by a resistive treatment, the power flow at the walls is distributed according to the distribution of pressure on the walls. The pressure is, in a "live" room overwhelmingly determined by the reverberant field. That the distribution of power flow into the walls need have no relation to the characteristics of the direct field, can be further demonstrated by considering the results of a patchy distribution of wall absorption on otherwise rigid walls. The power can flow only into the absorptive patches.

In some circumstances, that part of the reverberant sound field responsible for the power flow to the walls may be quite negligible compared with the standing-wave component. If, then, the analysis is really concerned with the total energy density, the errors incident to the false assumption may be small. However the proper first approximation in this case is that the power flow is negligibly small in the reverberant field, although nonvanishing.

b) The image field

Consider a tank which is a rectangular parallelepiped with walls either rigid or "pressure release". However assume that there is some attentuation in propagation through the fluid so that the sound pressure does not build up to indefinitely large amplitudes. In this special case, one can represent the total sound field as the superposition of the direct field of the source and the direct fields of an array of images

$$\psi_t = \psi_d + \Sigma \psi_i \quad , \tag{7}$$

both source and images being located in an infinite medium.

The procedure is a familiar one in acoustics (for a detailed discussion see Cremer, 1948, Chap. 3); the array of images is indicated schematically in Fig. 2. (Such a representation by images appears to be precisely correct only in the case of perfectly reflecting plane walls, as postulated here.) (See Morse and Bolt, 1944, section 53.)

The sound field near the source which is generated by the images alone (source off) must by definition be identical with the reverberant field defined in the previous subsection. This sound field, called here the "image field," is thus equivalent to the reverberant field generated by sources on the tank wall, in the special case where the image concept is valid.

The special case is extremely restrictive, but it appears so useful that many analyses use image considerations in cases where the concept is not strictly valid, for example when the walls of the tank are slightly absorptive. Quantitative analyses of the error not being available; one can only hope that the results retain some pertinence.

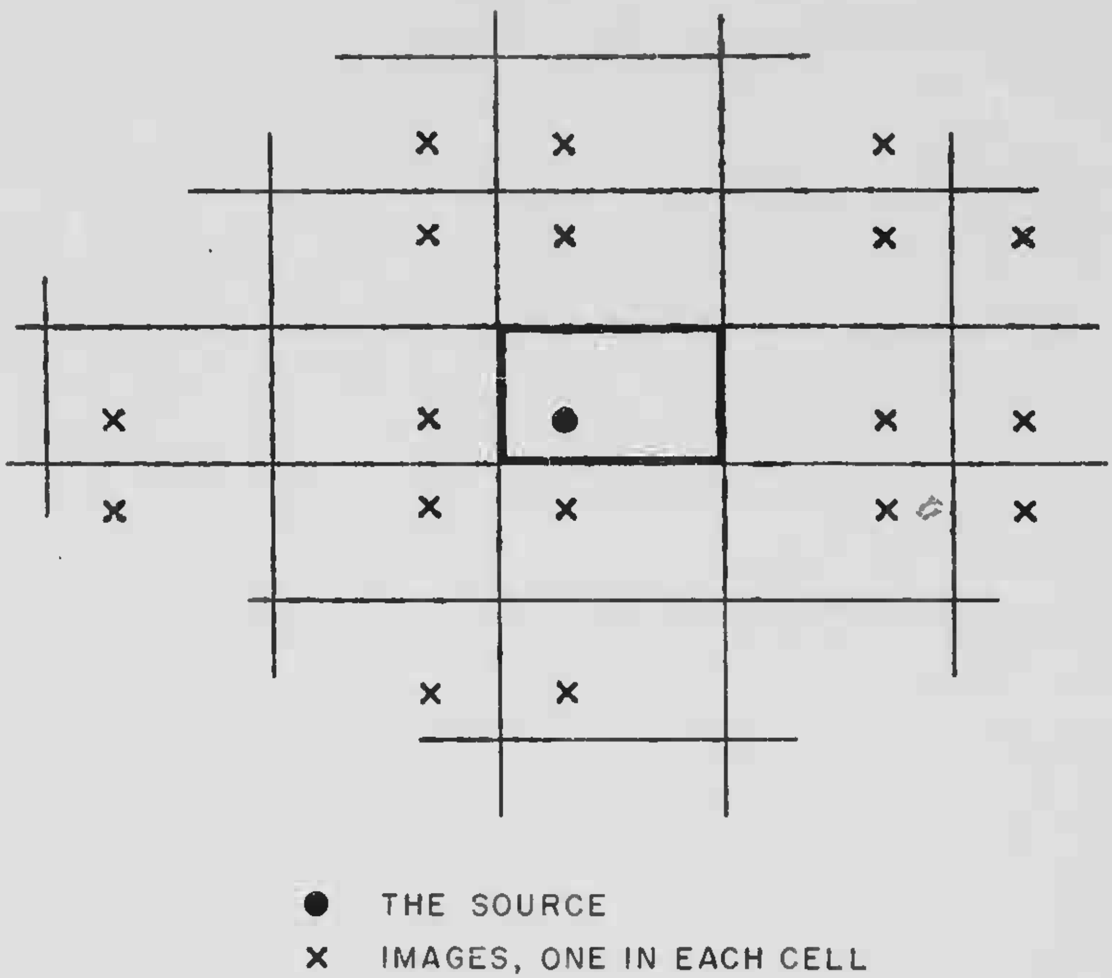


FIG. 2 SKETCH OF LOCATION OF IMAGES ABOUT A SOURCE IN A RECTANGULAR TANK

Nevertheless the image concept is a useful basis for discussing the significance of the earlier distinction between direct and reverberant fields. The distinction is based on the hope that, as we probe the sound field in different parts of the room, we will find no singularities which cannot be attributed to the direct field; the image field, we hope, will be fairly homogeneous everywhere in the room. The field of each image has its peculiarities to be sure. However, if a particular image is not near the boundary of the room, there will be many others at about the same distance but located in a variety of directions; we expect this variety to be sufficiently random that the total image field will be quite homogeneous.

These expectations and hopes are most likely to fail when the source is near a wall (so that the nearest images are close) and when the source (and therefore each image) is directive. If, because of the source's directivity and its closeness to a wall, the direct field is not negligible with respect to the reverberant field at any point on the room's walls, then the field of at least one of the nearest images will also stand out above the rest of the image field at that point.

c) Room reaction on the source

In transducer calibration, it is important to determine whether the existence of a reverberant field, in addition to the direct field, will react on the source and affect its operation. The question may be phrased: What is the acoustic impedance presented to the transducer and how does it affect the transducer's operation?

The answer to the second half of that question may be very hard to come by in the case of a multi-element, possibly nonlinear, array which is well coupled to the medium. One should probably, therefore, try to keep the reaction small at all times.

A number of analyses have been made for the effects of nearby surfaces (either hard or soft) on the sound power output of some types of sound sources (Waterhouse, 1955, 1958; Thompson and Junger, 1961). They assume that the reaction does not disturb the relative motions of elements of the transducer, an assumption that may be dangerous in some practical cases.

Their results generally confirm common engineering practice (Beranek, 1954, section 10.14) in some important practical cases.* In the case of a simple source (monopole), wall effects are relatively unimportant for distances greater than $\lambda/4$, where λ is the wavelength at the mid-frequency of the sound output.

The effects upon more complicated directive sources (dipoles, etc.) cannot be so simply summarized. Greater separations seem to be required if a lobe of the directivity pattern is pointed at the wall but not otherwise. (Of course it is unlikely that an experimenter investigating the characteristics of a directive sound source would point it at the nearest wall!)

---

*It is interesting to note that Beranek's procedures amount to a redefinition of the direct field to include the fields of both the source and the nearest images, and therefore a consequent redefinition of the reverberant field. Yet no corresponding modification is made in the room constant although consistency seems to indicate the need for one. The question requires further investigation.

We sketch below several possible approaches to the question of room reaction on complicated sources, and draw some tentative conclusions. The first approach is based on the image description of the sound field. The second approach is based on the direct-reverberant description of the total sound field and assumes that means are available for estimating the strength of the reverberant field.

d) Image considerations

When the walls are sufficiently reflecting that the image picture of the reverberant field can be viewed with some confidence, the question of reaction on the source can be translated into a question of interaction between one element (the source) and all others (the images) in a large array. Unfortunately the array is very large, having no limit. In the presence of absorption the strengths of the individual images decrease with increasing distance from the source, but the validity of the image concept decreases as well, as has been remarked above.

If the source were close to one wall, so that the field of the nearest image were particularly large, one might be satisfied (or at least, reassured) if the reaction on the source due only to this nearest image were small. The analysis necessary to express even this primitive criterion quantitatively would not be simple. An experimental approach - using two transducers, driving the "image" and measuring reaction on the "source" - seems somewhat impractical. The experiment must be run in a larger tank where the room reaction on both transducers is known to be small. (However scaled model experiments could probably be made.) The theoretical approach, analyzing the diffraction around the source of sound from the nearby image, promises difficulties of awesome complexity for most practical transducers.

e) Reverberant field considerations

Let us now turn from the image picture to the "direct-reverberant" description of the total field and consider what conclusions about reaction upon the source can be drawn from it. The complex radiation impedance presented to a surface S_o is proportional to the integral

$$Z_o \propto \int_{S_o} p \; v_n^* \, d\sigma \qquad (8)$$

where p is the sound pressure and v_n^* is the complex conjugate of the normal component of velocity on S_o, both written in complex notation. [See Appendix I, Eq. (10).]

Now the total pressure p is the sum of two terms

$$p = p_{dir} + p_{rev} \quad , \qquad (9)$$

the first of which, the direct-field pressure, has been defined above as the pressure that would be observed in free space. (If S_o includes only part of the total active surface - for example, just one element, the p_{dir} includes contributions from the other active elements.) It is evident that the effect of the reverberant field upon impedance will be negligible if the reverberant pressure is "sufficiently negligible" in comparison with the direct pressure. But, how small is negligible? Unfortunately, the answer depends in detail upon the transducer design; general conclusions can hardly be formulated.

Consideration of the relative phase of the pressure components is pertinent to the problem of estimating the reaction. The phase of the reverberant pressure (for pure tone excitation) will vary from point to point over the surface of the transducer. At any one

transducer element, it may be either in phase with the velocity (affecting the real part of the acoustic impedance) or in phase quadrature with velocity (affecting the imaginary part of the acoustic impedance). In some transducers, particularly superdirective arrays, proper operation may require that the reverberant pressure have negligible effect on both the real and the imaginary parts of the impedance. This is obviously a more severe restriction than that resulting from a criterion of negligible effect on the magnitude of impedance.

We outline in the next section some procedures of geometrical acoustics which will usually be adequate for estimating the level of the reverberant pressure. Procedures for estimating the level of the direct field on the face of the transducers are notably lacking. Analytic solutions have been found in a few special cases. (See Hanish, 1960a, for a review, and Junger, 1960a.) No suitable engineering approximations are apparent. Experiments have demonstrated the large fluctuations that exist in pressure magnitudes (see various Appendices to Klein, 1960a), but information on the phase of the acoustic impedance was not reported.

It is revealing to consider reasons why radiation impedance is not more of a problem for measurements of power output into air. First, because of the greater impedance mismatch between source and medium in the air-load case, variations in magnitude and distribution of radiation loading will not usually affect either the magnitude or distribution of source strength (surface velocity). Secondly, for a pure-tone signal, variations from point to point over a large source (several wavelengths) will lead to a cancellation in forming the net reverberant reaction. Thirdly, with a small source, the ratio of direct to reverberant pressures on the surface is larger. Finally,

most signals of interest in architectural acoustics are reasonably broad bands of noise. In some cases of multiresonant systems, such as a room is, it has been found by analysis that the average radiation impedance for a band of noise equals the characteristic impedance of an infinite space (free-field loading).

The generality of this last point appears not to have been established, nor is information available on the average mutual radiation impedance of two sources driven by bands of noise. These questions are of great importance to better understanding of reverberant water tank acoustics.

B. SOUND FIELDS IN A TANK: THEORETICAL PREDICTIONS

In this section we summarize some of the available theoretical predictions for sound fields in a room.

1. Results of Geometrical Theory

The geometrical theory of room acoustics is a high-frequency approximation based on assumptions of (1) essentially uniform, diffuse distribution within the room of the energy of the reverberant field, and (2) essentially equal probability of propagation of sound in all directions from a point. (Morse and Bolt, 1944, section 6). The results of the geometrical theory are therefore average, statistical formulae expressed in terms of average overall characteristics of the tank. (For a general reference giving derivations of these results, see Morse and Bolt, 1944.)

Most of the difficulties and much of the literature associated with the geometrical theory are concerned with the calculation of the "average sound absorptivity" of the tank from characteristics of the materials of which it is made. These difficulties have not been

entirely resolved, and probably never will be. However they have been clearly stated and discussed in the recent literature (Young, 1959a,b). We shall sidestep these problems here, and speak only of the overall Sabine sound absorption A, in sabins, of the tank.

The Sabine absorption A is related to the rate of decay of pressure in the tank after the source is turned off. If the decay rate is K neper per unit of time [pressure≈exp(-Kt)], then

$$K = cA/8V \text{ neper/time unit,} \tag{10}$$

where V is the volume of the tank. For translation purposes we note:

reverberation time: $T_r = 3/K \log_{10}e = 6.9/K$ time units

decay rate: $D = 20K \log_{10}e = 8.7K$ db/time unit

Thus, the values: $T_r = 1$ sec, $D = 60$ db/sec, $K = 6.9$ neper/sec, describe the same decay.

a) Power-pressure relations

In a steady-state condition, the power input to the room W, the average energy density E, and the root mean square sound pressure $\overline{p}$ are related by:

$$W = 2KV = cAE/4$$

$$E = \overline{p}^2/\rho c^2 \quad ,$$

and thus $$\overline{p}^2 = 4W\rho c/A \quad . \tag{11}$$

In these equations, $\overline{p^2}$ is the average of the square of instantaneous pressure, with the average taken both in time and space throughout the room. The energy density E is similarly averaged. The mean square pressure at any single point may differ from the average value, in cases by as much as 10 db or more.

The variations from point to point of rms pressure are of two kinds. One is a response irregularity discussed below under "Results of the Wave Theory"; the average size of these fluctuations in level is $\pm$ 5 db and the average spacing between maxima is 0.7λ. The second kind of variation results from "pressure-doubling" near hard walls and "pressure-cancellation" near pressure-release walls.

As Waterhouse (1955) has shown, within a distance λ/4 of a hard wall the average pressure rises sharply to reach $\sqrt{2}$ times its value at positions more distant from the wall; this result is valid for broad-band noise as well as tones if λ is evaluated at an arithmetic mean frequency. Analyses for pressure-release walls have not been done; as a guess, one would expect a complementary behavior: the average pressure starts decreasing at a distance λ/4 from the wall to reach zero at the wall.

As a result of this "pressure-doubling" effect the average pressure p_c in the central part of the tank will be less than the average over the whole volume, $\overline{p}$, which enters in Eqs. (11). Based on Waterhouse's analyses and our estimate of behavior at a pressure release wall, this effect is, to a first approximation,

$$\frac{\overline{p^2}}{p_c^2} = 1 + \frac{\lambda}{8} \frac{S_h - S_r}{V}$$

where S_h is the area of hard walls and S_r is the area of pressure-release walls. Subject to the usual approximation that

$$S = S_h + S_r \approx 6V^{2/3} \quad ,$$

the equation can be rewritten

$$\frac{\overline{p^2}}{p_c^2} \approx 1 + \frac{3(2h-1)}{4\mu} \tag{12}$$

where h = fraction of wall area which is hard;

$\mu = V^{1/3}/\lambda$ is a frequency parameter.

In these considerations we have implicitly assumed that all walls are either hard or pressure-release, that is, that the phase shift on reflection is either 0 or π. However, especially in a tank of water, it is quite conceivable that intermediate cases are important. It is almost certain that the preceding considerations are invalid in such cases. There probably exist situations in which the average value of pressure stays constant as the measurement point approaches a wall. These questions should be investigated in some detail.

Another question of importance in estimating reverberant field reaction on the source is whether the average reverberant pressure on the transducer's face may be larger than the average pressure out in the tank, due to a diffractive "pressure-doubling" effect. The question should be subjected to careful analysis; however qualitative considerations suggest that no such enhancement of pressure exists.

Consider a point on a large rigid scatterer of arbitrary convex shape in a reverberant sound field. Describe the reverberant field as a large number of plane waves, approaching from all directions. The waves approaching the point from the 2π ster-radians of solid angle which "illuminate" it directly, will be subject to the "pressure-doubling" effect (Waterhouse, 1955). But the point will be shielded from the waves approaching from the other 2π ster-radians. Therefore, in the high frequency limit, the average pressure at any point on the scatterer equals the average pressure at any point away from it.

b) Acoustic ratio

A relationship of great interest to the present problem is the "acoustic ratio" R, which is often defined as the ratio of the reverberant sound energy density to the energy density of the direct sound field of the source. However, since the acoustic ratio is then never used except in the far field of the source, where $E \sim p^2$, this definition is equivalent to the one we will use:

$$R \equiv p_{rev}^{2}/p_{dir}^{2} \quad , \qquad (13)$$

the square of the ratio of reverberant and direct sound pressures. [Stroh, 1959, gives a summary with experimental data.]

The value of R depends upon position in the room because of the variations of the direct field pressure. In the far field of a directive sound source, the mean square direct pressure is

$$p_{dir}^{2} = WD\rho c/4\pi r^{2} \qquad (14)$$

where W is the power output, D is the directivity factor at the observation point, and r is the distance from source to observation point.

Now, the reverberant pressure at a particular point is variable both with frequency and position. These fluctuations were discussed in the last sub-section. However, on the average, the reverberant pressure is given by Eq. (11). The resulting combined equation for average acoustic ratio in the far field, but not near a wall, is:

$$R = 16\pi r^2/AD \quad . \tag{15}$$

c) Mean-free path

Finally, we note that the fundamental measure of the size of a tank in geometrical acoustics has been the mean free path, ℓ_c. When a sound field is perfectly diffuse, the average path length between "collisions of the sound rays" with the walls is (Kosten, 1960):

$$\ell_c = 4V/S \tag{16}$$

where S is the total area of the wall. Since in many tanks

$$S \approx 6V^{2/3} \quad ,$$

we have

$$\ell_c \approx (2/3)V^{1/3} \quad . \tag{17}$$

2. Results of Wave Theory

In the wave theory of room acoustics, the sound field is described in terms of a summation of responses in the individual natural modes. The most useful results are approximations based on an assumed statistical randomness in the characteristics of these modes. Thus, for example, the marked degeneracy in the

frequencies of the modes which occurs in a perfectly cubical room must be considered as an abnormal situation not comprehended by the usual theoretical approximations.

In wave theory, dimensions are conveniently measured in units of a characteristic length

$$L_c \equiv V^{1/3} \approx 1.5\ell_c \quad , \tag{18}$$

which is typically about 1.5 mean free paths [Eq. (17)]. Similarly frequency can be measured in units of a characteristic frequency

$$f_c \equiv c/L_c \quad , \tag{19}$$

so that the frequency f is replaced as a variable by a frequency parameter

$$\mu \equiv f/f_c = L_c/\lambda \quad , \tag{20}$$

where λ is the wavelength of sound.

a) Modal density

The density of modes - the number n of modes whose frequencies lie in a bandwidth of one characteristic frequency - averages (Morse and Bolt, 1944, Eq. 3.4)

$$\frac{dn}{d\mu} = 4\pi\mu^2 + \frac{\pi}{2}\frac{S}{L_c^2}\mu + \ldots \text{ modes per bandwidth } f_c \quad . \tag{21}$$

The second term becomes negligible at high frequencies. The equation can be rewritten as

$$\begin{aligned}\frac{dn}{d\mu} &= 4\pi\mu^2\left(1 + \frac{1}{8}\frac{S\lambda}{V} + \ldots\right)\\ &\approx 4\pi\mu^2\left(1 + \frac{3}{4}\frac{\lambda}{L_c} + \ldots\right)\\ &\approx 4\pi\mu^2\left(1 + \frac{3}{4\mu} + \ldots\right) ,\end{aligned}$$

where the approximation $S \approx 6V^{2/3}$ was used in the last two expressions. (These formulae do not depend on the character of the walls, hard or soft, so that they are valid for the water tank with one pressure release surface. Of course the precise values of the modal frequencies are affected by the character of the walls.)

Each mode behaves like a damped mass-spring system, its response as a function of frequency being a single peak whose bandwidth depends on the amount of sound absorption. Let it be assumed that each mode decays with the same decay rate, K nepers per unit of time. Then the bandwidth of each mode is

$$\Delta = K/\pi \text{ cycles per time unit.} \tag{22}$$

Now in the wave theory of room acoustics the "high-frequency" condition is reached when many modes participate in the response at any one frequency. Thus we must inquire how many modes have natural frequencies in any band Δ. The average number of modes per bandwidth Δ is found from Eqs. (21) and (22) to be

$$\begin{aligned}N &= (\Delta/f_c)\, dn/d\mu\\ &= \frac{4Kf^2}{f_c^3}\left(1 + \frac{1}{8}\frac{S\lambda}{V} + \ldots\right) ;\end{aligned} \tag{23}$$

or, when Eq. (10) relating K to the Sabine absorption A is valid,

$$N \doteq \frac{1}{2}\frac{A}{\lambda 2}\left(1 + \frac{1}{8}\frac{S\lambda}{V} + \ldots\right) \quad .$$

The expression for N assumes a very neat form when one introduces the average Sabine coefficient for the tank,

$$\bar{a} = A/S \quad , \tag{24}$$

and makes the approximation $S \approx 6V^{2/3} = 6L_c^2$. Then one gets

$$N \approx 3\bar{a}\mu^2(1 + 3/4\mu + \ldots) \quad , \tag{25}$$

where $\mu = L_c/\lambda$.

b) Response irregularities

The steady-state sound pressure produced in a tank by a pure-tone source, at another point, fluctuates with variations either in frequency or in the position of the observation point. At low frequencies these fluctuations in total pressure are related to the corresponding fluctuations in the strength of individual modes. At high frequencies the fluctuations arise from unavoidable variations from the statistical average value of the actual spacing of modal spacings. We summarize here some of the major results of existing analyses* [Bolt and Roop, 1950; Schroeder 1954; Bolt and Schroeder, 1958].

---

*We do not draw on the analysis of Doak (1959) despite his apparently good check between theoretical and experimental results. As noted earlier, Doak has erroneously identified the reverberant field as a purely standing-wave field. Moreover, in his view, the values of local minima in the total pressure are set by the direct field. Therefore it appears that the error is intimately involved in the analysis.

At high frequencies the average range of fluctuation is about $\pm$ 5 db. The average frequency interval between adjacent maxima equals the decay rate $K=6.9/T_r$ where T_r is the reverberation time [Eq. (10)]. (If T_r is in seconds, the spacing is in cycles per second.) The average spacing between maxima, taking all orientations in the room, equals about 0.7λ. These high-frequency results are independent of all characteristics of the room except for the decay rate (reverberation time).

At low frequencies the average range of fluctuation increases markedly. The spacing between maxima is about the same, 0.7λ. However the frequency interval between maxima increases, becoming equal to the average interval between modal frequencies (the reciprocal of modal density, Eq. (21)).

c) Criterion for high-frequency behavior

A particularly important result of the analyses of response irregularities is the criterion for the transition frequency between low and high frequency regions:

> High frequency behavior requires both (1) that the number of modes per modal bandwidth [N in Eqs. (23), (25)] be
>
> $$N \geq 10 \,, \tag{26}$$
>
> and (2) that the individual normal modes be sufficiently perturbed that the phases of their response amplitudes are randomized (Bolt and Schroeder, 1958).

This criterion for high frequency behavior appears also to be sufficient guarantee that the basic assumptions of the geometrical theory be valid. If the criterion is not satisfied, one cannot expect the predictions of the geometrical theory to be accurate.

The quantitative part of this criterion can be transformed, by Eq. (25), into the condition

$$\mu > 2/\bar{a}^{1/2} \quad , \tag{27}$$

where $\mu = L_c/\lambda = V^{1/3}/\lambda$.

The qualitative part of the criterion, randomness in modal responses, draws attention to the need for (1) a distribution in patches of the absorbing material, and (2) perturbations ("bumps") in the shape of the tank walls. A semi-quantitative "index of randomness" has been derived by Morse and Bolt (1944, section 50). It indicates that randomness is achieved by either (1) about 40 or more bumps which are about $\lambda/2$ high and λ^2 in area, or (2) about 100 or more patches of absorptive material which have each an area λ^2.

Now it may be physically impossible to install so many bumps and patches. Indeed, if we say that the available wall area is $S \approx 6V^{2/3}$ and that the bumps or patches (of area λ^2) will cover 50% of S, minimum frequencies for achieving the required number are:

$$\begin{aligned} &\text{for 40 bumps} \quad , \quad \mu > 5 \\ &\text{for 100 patches} \ , \quad \mu > 8 \quad , \end{aligned} \tag{28}$$

where $\mu = L_c/\lambda = V^{1/3}\lambda$.

The consequence of inadequate randomness, exhibited in some experimental measurements of transmission irregularities (Bolt and Schroeder, 1958), is that low-frequency behavior may persist to frequencies one or more octaves above the transition frequency given in Eqs. (26) and (27).

We have in this discussion of the criteria for validity of the equations of architectural acoustics, reached the point where quantitative predictions tend to disappear in favor of intuitive judgements and proof by experiment.

C. SUMMARY

1. Section A: Fundamental Concepts

First, the concepts of near and far fields of a transducer in free space are reviewed. Three conditions [Eqs. (1)-(3)] are stated which are believed to be necessary and sufficient conditions for the existence of all the common attributes of the "far field". However some of the attributes of the "far field" may obtain in regions where the conditions are not met, in the case of some sources. Therefore it appears that the degradation in the near field of far-field attributes is intimately related to the configuration of the source. Quantitative general information on such relationships is not available since analyses of the near field have been carried out only for a few specific source configurations.

Second, the fundamental concepts of "room acoustics" are surveyed with the particular problem of water-filled tanks in mind. These concepts are the direct and reverberant fields, images, and interactions between room and source. One traditional aspect of the "reverberant field" -- that it is a standing wave field with no net power flow -- is found to be faulty. (The resulting errors will be shown in later sections of this report to invalidate some proposed measurement schemes.)

Two aspects of the reaction of a tank onto a transducer were discussed and criteria for negligible reaction sought. The effect of a nearby wall on power output from a non-directive source is minimized

by maintaining at least $\lambda/4$ separation. On the basis of scanty available information, it is estimated that no greater separation is required with a directive source, as long as the beam is not directed at the nearest wall.

The effect of the reverberant field upon the radiation loading of the transducer is minimized if the reverberant pressure is less than the direct (or "free") field pressure. However if one requires that neither the resistive nor reactive components of impedance be affected, this criterion may require more information about the near-field than is now generally available.

2. Section B: Sound Fields in a Tank; Theoretical Predictions

Pertinent quantitative predictions, resulting from the geometrical and wave theories of architectural acoustics, are summarized.

The results of the geometrical theory include the means for predicting (1) total power output from pressure measurements in the reverberant field [Eq. (11)] and (2) the range of distances from the source in which the direct field will exceed the reverberant [Eq. (15)].

The results of the wave theory include some criteria for the validity of the geometrical theory [Eqs. (26) and (27)], although the question of randomness of the reverberant field cannot be answered completely in quantitative terms.

The wave theoretical analyses of transmission irregularities indicate:

(1) The range of frequencies which must be included in a "noise" signal if smooth response at one position is to be expected (several times the average interval $6.9/T_r$ between maxima);

(2) The size of volume over which pressures must be averaged if smooth response to a pure tone is to be expected (several times the average distance 0.7λ between maxima); and

(3) The range of fluctuation (an average of $\pm$ 5 db) of pure-tone reverberant-field pressure about its average value.

From the last result one may estimate how much of a safety factor on acoustic ratio [Eq. (15)] is required for direct-field measurements.

III. TANK DESIGN: ENGINEERING CONSIDERATIONS

In this chapter we use the theoretical predictions of room acoustics to derive restrictions on the design of a calibration tank.

The type of transducer we consider has the following characteristics:

maximum linear dimension:	$2a \approx 4\lambda$
active area of source:	$S_o \approx 16\lambda^2$
maximum (on-axis) directivity factor:	$D_{max} \approx 10$ to 10^2

We consider more or less reverberant tanks and take the average Sabine absorption coefficient $\bar{a}$ and the frequency-size parameter $\mu \equiv V^{1/3}/\lambda$ as variables. Throughout we use the approximation for tank surface area: $S \approx 6V^{2/3}$. We are especially interested in tanks whose volume V is not more than about $10^4\lambda^3$, i.e., $\mu = 22$.

1. Where is the Far Field?

Consider the transducer placed in a free field or a truly anechoic tank. The restrictions that the distance r to an observation point lie in the far field were given as inequalities in Eqs. (1), (2), and (3). In Fig. 3 (top) we plot these restrictions, replacing the inequality by an equality; thus we plot:

$$r/\lambda = a/\lambda \tag{1}$$

$$r/\lambda = \pi(a/\lambda)^2 \tag{2}$$

$$r/\lambda = 1 \tag{3}$$

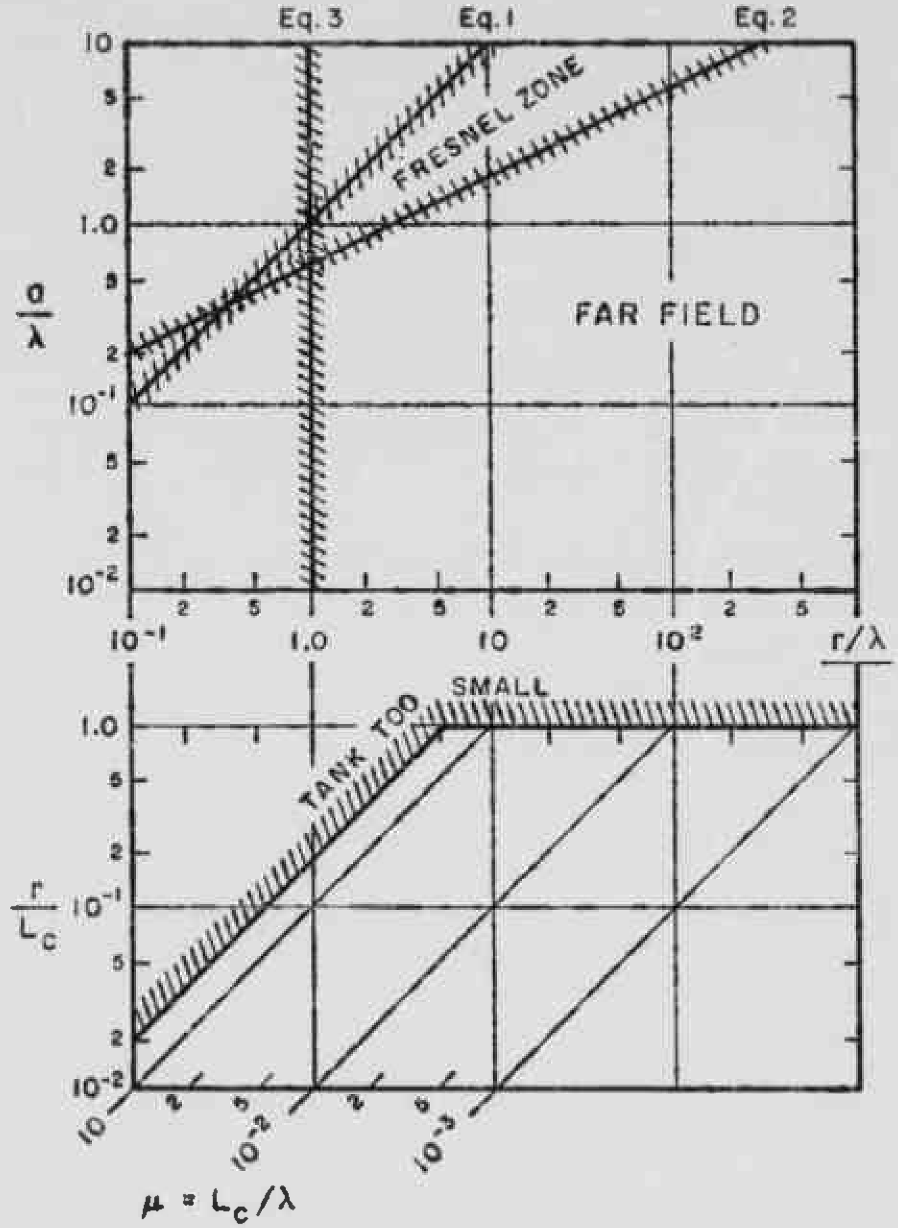


FIG. 3 FAR FIELD BOUNDARIES

Only a point well to the right of all the plotted boundaries will be surely in the transducer's far field.

In the bottom half of Fig. 3 we show the relation:

$$\mu(r/L_c) = r/\lambda$$

where $L_c = V^{1/3}$, $\mu = L_c/\lambda$. Since L_c is roughly a typical dimension of the tank it is physically impossible for (r/L_c) to exceed unity by very much. Indeed, if we wish to measure in all directions from a transducer, we require

$$r/L_c \lesssim 1/2 \quad .$$

The boundary marked "tank too small" on Fig. 3 is compounded from the lines $r/L_c = 1$ and $\mu = 5$, the latter being the minimum size for which diffusion can be expected [see Eq. (28)].

For the typical transducer, we have assumed

$$a/\lambda \approx 2$$

so that the far field lies at

$$r/\lambda \gtrsim 12 \quad . \qquad (29)$$

If the observation point is to be within the confines of a tank, the frequency and tank size must be such that

$$\mu \gtrsim 24 \text{ wavelengths per average side,}$$
$$V \gtrsim 1.4 \times 10^4 \lambda^3 \quad .$$

2. When will the Reverberant Reaction on the Source be Small?

This question is one of the most difficult to answer quantitatively. [See discussion in "Room reaction on the source," above.] An answer requires predictive knowledge of the near field which we just do not have. However we can hazard some guesses.

Let us try to guess when the reverberant field pressure may significantly affect the real part of the radiation loading on individual elements of the transducer. We assume that the transducer is not near a wall (i.e., not within a distance $\lambda/4$). We try to put numbers to the earlier qualitative discussion.

The real part of the radiation impedance accounts for the power flow. The power flow (described by the real part of the Poynting vector) is continuous from near field to far field [see Appendix I, particularly Eq. (6)]. Thus the average intensity on the active surface of the transducer is

$$\bar{P} = W/S_o$$

where W is total power radiated and S_o is the active area. But how is this distributed over the area S_o? The distribution is probably not uniform, but it may not be far from uniform. Let us guess that the minimum intensity is

$$P_{min} \approx W/2S_o \quad .$$

The next question is how to relate this direct field intensity to a direct field pressure, that is, what is the value of the acoustic conductance. In the middle of a very large transducer, which

is not concave, the conductance is $(1/\rho c)$ (Lax and Feshbach, 1947). Therefore we guess that the component of pressure in phase with velocity is

$$p_P^2 \approx P\rho c$$

whence, for the estimated minimum intensity,

$$p_P^2 \approx W\rho c/2S_o \quad .$$

This estimate must be compared with the estimate for the reverberant pressure (section "Power-pressure relations" and Eq. (11), above) which is, on the average:

$$\overline{p}_{rev}^{\,2} \approx 4W\rho c/A$$

$$\approx 4W\rho c/S\overline{a} \quad .$$

We allow for spatial fluctuations of ± 5 db:

$$\hat{p}_{rev}^{\,2} \approx 3p_{rev}^{\,2} \approx 12W\rho c/S\overline{a} \quad .$$

Some manipulations of the two estimates of pressure leads to the ratio

$$p_P^2/\hat{p}_{rev}^{\,2} \approx S\overline{a}/24S_o$$
$$\approx \mu^2\overline{a}/4(S_o/\lambda^2) \quad , \qquad (30)$$

where we have used the familiar estimate

$$S \approx 6V^{2/3} = 6\mu^2\lambda^2 \quad .$$

For small change in radiation resistance, we must require that this ratio be large. For example, if the ratio is 4, then

$$\hat{p}_{rev} \approx 0.5\ p_p$$

$$\overline{p}_{rev} \approx 0.3\ p_p \quad .$$

In this case we must expect fluctuations in power radiated (and thus in radiation resistance) which on the average are in the range $\pm 30\%$ and occasionally could be 50% or more.

With this criterion for small change in radiation loading, the minimum conditions on the tank design are expressed by

$$\mu^2 \overline{a} \gtrsim 16(S_o/\lambda^2) \quad . \tag{31}$$

As a typical example, take $S_o = 16\lambda^2$, $\overline{a} = 1/4$; the tank size must be

$\mu = 32$ wavelengths per side.

Because of the nature of the guesses and estimates we have perforce used, we cannot express great confidence in the results. We have tried to make "reasonable" guesses throughout, so that the result must not be considered a conservative estimate of the conditions for small reaction.

3. When will the Reverberant Field Predominate on all Tank Walls?

In order to be able to calculate power output from the average sound pressure by Eq. (11):

$$\overline{p}^2 = 4W\rho c/A \quad , \tag{11}$$

we must require that the reverberant field predominate over all of the tank walls. If the direct field were to predominate at some point, the absorptivity of that section of the wall would have to be given greater than average weight in the determination of the average absorption A in Eq. (11). In other words, the effective value of A would depend upon the orientation and directivity of the particular source transducer.

We start from Eq. (15) for the average acoustic ratio R, using $A = S\bar{a}$ and $S \approx 6L_c^2$, and derive

$$R D \bar{a} \approx 8(r/L_c)^2 \quad , \tag{32}$$

an expression valid only in the far field of the source. [Note: $16\pi/6 = 8.3 \approx 8.$] Now a nondirective source, D = 1, should obviously be positioned roughly in the center of the room so that the maximum distance r to any wall will equal about $(L_c/2)$. However, highly directive sources should be positioned asymmetrically with the greatest possible distance between source and wall in the direction of the main beam. (Of course, the source must not be so close to the wall as to be affected by "pressure-doubling" effects' see "Room reaction on the source," above.) In such a case, then, $(r/L_c) \approx 1$. We require, therefore, that

$$R \approx 8/D_o\bar{a}$$

be large, where D_o is the maximum directivity factor.

How large must R be? We must take cognizance of two factors. First, wall effects lead to a doubling of p_{rev}^2 at the wall in comparison to the average value, if the wall is hard (see "Power-pressure

relations," above). Secondly, the average peak in the spatial fluctuation of p^2 corresponds to a factor 3, that is ± 5 db in level (see "Response irregularities," above).

A reasonable compromise might be

$$R > 4;$$

then the average reverberant squared-pressure will exceed the direct squared-pressure by a factor of 8, but there will be points (about every $\lambda/2$) where the factor will be about 3 or even less.

With this compromise condition, $R > 4$, we readily derive the condition on $\bar{a}$:

$$\bar{a} \lesssim 2/D_o \quad . \tag{33}$$

The table below shows some typical values.

TABLE I

MAXIMUM ALLOWABLE ABSORPTION FOR REVERBERANT FIELD TO PREDOMINATE AT WALLS

Directivity factor on axis, D_o	Directivity index	Maximum allowable absorption, $\bar{a}$
10	10 db	0.2
30	15 db	0.07
100	20 db	0.02

It appears that very little absorption can be tolerated if a reverberant condition is to be achieved on all walls with a highly directive source.*

4. Where will the Direct Field Predominate?

We inquire here into the conditions that the direct field pressure dominate the reverberant field pressure so that we may hope to determine a free-field directivity pattern from tank measurements. We consider as typical a transducer whose maximum dimension is $2a \approx 4\lambda$.

Reasonable estimates can be formed, when the observation point is in the far field, by using Eq. (15) for the acoustic ratio. This ratio of squares of the average reverberant pressure and the direct pressure is

$$R = 16\pi r^2/AD \quad . \tag{15}$$

We transform the equation by

$$A = S\bar{a} \approx 6L_c^2\bar{a} = 6\mu^2\lambda^2\bar{a}$$

into the form

$$\mu^2\bar{a} \approx 8(r/\lambda)^2/RD \quad . \tag{34}$$

[Compare Eq. (32).]

* We pose the hypothetical question whether the situation cannot be rectified by the installation of highly reflective, non-absorptive scatterers in the path of the main sound beam.

Now the actual reverberant field pressure fluctuates from point to point and in the high frequency limit the average fluctuation in level is ± 5 db, or a factor of about 3 in squared-pressure (see "response irregularities," above). We choose here as a criterion for a borderline predominance of direct field

$$R \lesssim 1/3 \quad .$$

We require that this borderline condition be met at the -10 db points of the directivity pattern, where $D = D_o/10$. Thus only the main lobe of the direct field will stand out above the reverberant field. The combined criterion is, then,

$$RD \lesssim D_o/30 \quad .$$

From this criterion and Eq. (34) we determine the condition on tank characteristics:

$$\mu^2 \bar{a} \lesssim 240 \, (r/\lambda)^2/D_o \quad . \tag{35}$$

For specified values of $\bar{a}$ and D_o, this inequality [Eq. (35)] sets a minimum value on the ratio $\mu/(r/\lambda) = L_c/r$. Typical values are shown in Table II below.

TABLE II

MINIMUM VALUE OF $\mu/(r/\lambda) = L_c/r$ FOR MAIN DIRECT-FIELD LOBE (ABOVE -10 DB POINTS) TO DOMINATE REVERBERANT FIELD. (The values marked with a star are suspect because the direct field is predominant at some points on the tank wall.)

Directivity Factor on Axis, D_o	Absorption Coefficient $\bar{a}$	Minimum of $\mu/(r/\lambda)=L_c/r$
10	.1	15
10	.3	9*
30	.1	9*
30	.3	5*
100	.1	2*
100	.3	1*

If information about less important details of the directivity pattern were required (that is, for $D < D_o/10$), even more stringent conditions would result.

5. <u>Summary</u>

The results of these considerations of tank acoustics are summarized in Figs. 4 and 5. Each is pertinent to a transducer with:

$$\text{largest dimension:} \quad 2a = 4\lambda$$
$$\text{active area:} \quad S_o = 16\lambda^2$$

The largest dimension determines the boundary between near and far field parts of the direct field; the boundary is closer (r smaller) when <u>a</u> is smaller. The active area enters into our rough estimate

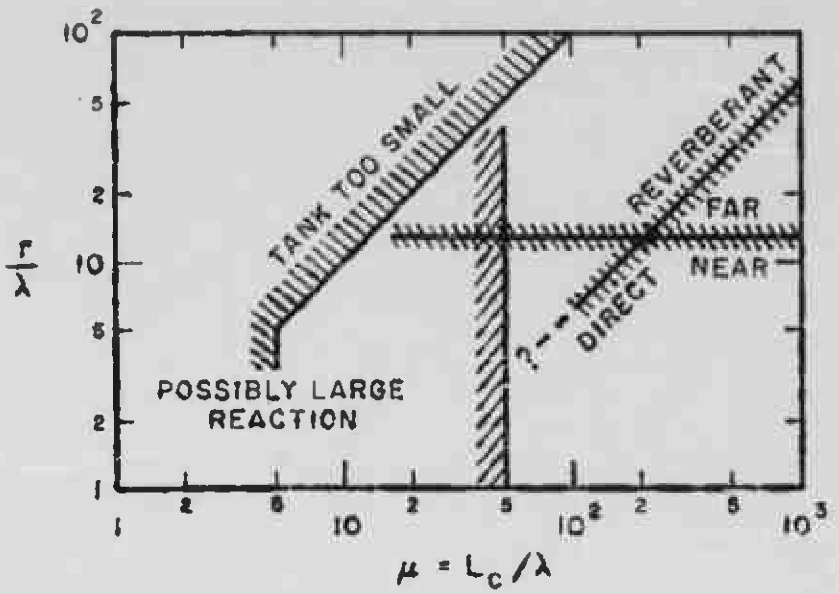
TANK AND TRANSDUCER CONDITIONS

(a) $2a = 4\lambda$

$S_a = 16\lambda^2$

$\bar{a} = 0.1$

$D_o = 10$



(b) $2a = 4\lambda$

$S_o = 16\lambda^2$

$\bar{a} = 0.25$

$D_o = 10$

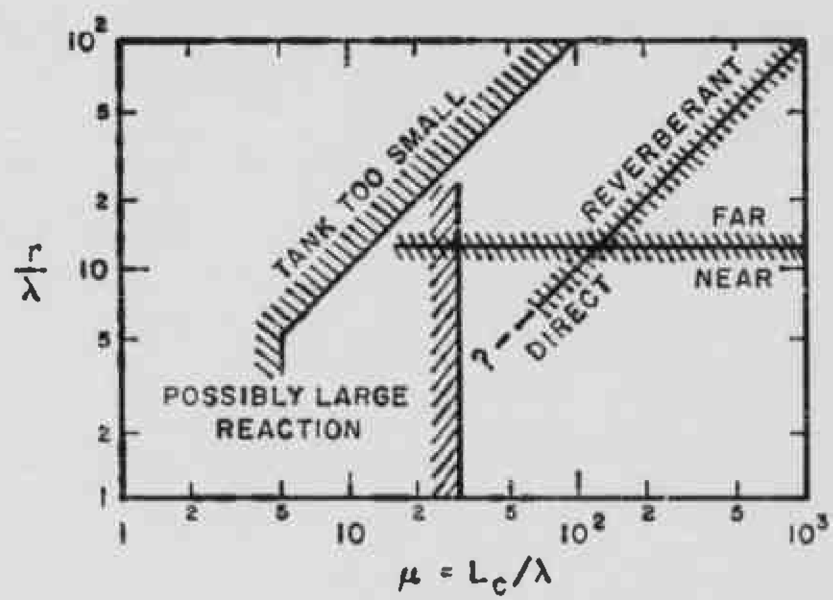


FIG. 4 ACOUSTICAL BOUNDARIES FOR MAXIMUM DIRECTIVITY FACTOR $D_o = 10$

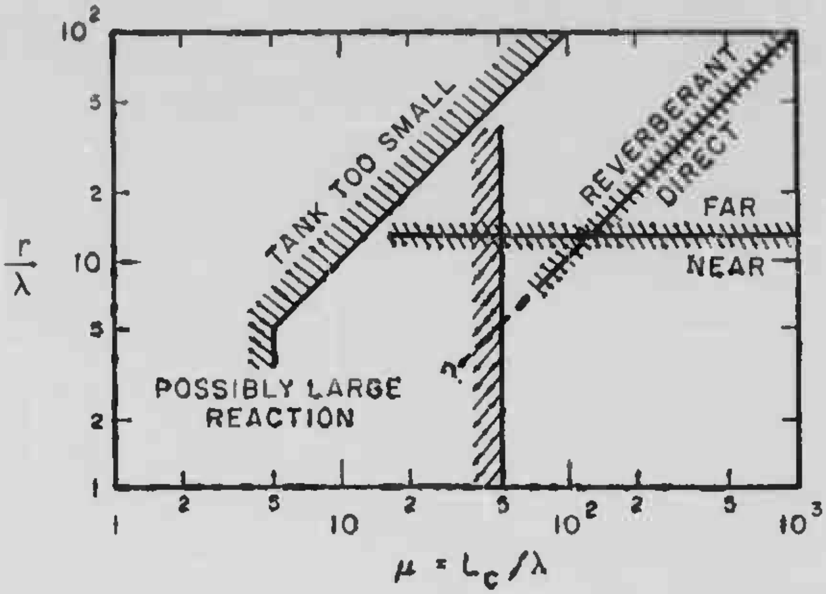
TANK AND TRANSDUCER CONDITIONS

(a) $2a = 4\lambda$

$S_o = 16\lambda^2$

$\bar{a} = 0.1$

$D_o = 30$



(b) $2a = 4\lambda$

$S_o = 16\lambda^2$

$\bar{a} = 0.25$

$D_o = 30$

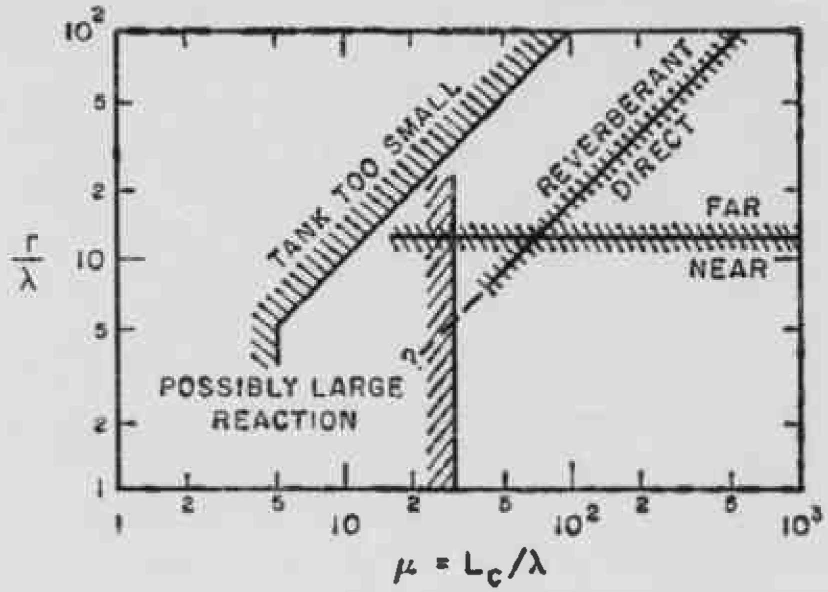


FIG. 5 ACOUSTICAL BOUNDARIES FOR MAXIMUM DIRECTIVITY FACTOR $D_o = 30$

of the conditions for a small room reaction on the source [part 2 of this chapter Eq. (31)]; if the area S_o is smaller, lower frequencies or a smaller tank (smaller μ) can be used without increasing the reaction.

Figure 4 is applicable to a transducer with maximum directivity factor $D_o = 10$ (directivity index = 10 db). In part A (top) we assume a value of the average Sabine absorption coefficient $\bar{a} = 0.1$; in part B (bottom) we assume $\bar{a} = 0.25$.

The boundary marked "tank too small" is defined by the lines $r = L_c$ and $\mu = 5$. To the left of $r = L_c$, the desired separation r is larger than the average tank dimension. To the left of $\mu = 5$, the tank size is probably too small to achieve diffusion [see Eq. (28)]. The other criterion for high-frequency behavior [Eqs. (26) and (27)] is less severe in this case.

The diagonal line marking a boundary between "reverberant" and "direct" is an estimate of the tank conditions for which the main lobe of the direct field will predominate over the reverberant field. However in those directions in which the direct field pressure level is more than 10 db below the value on the main lobe's axis, measurements closer to the transducer would be required for a reliable predominance. In any case, the estimate for the location of this boundary is poor for points in the near field.

We conclude from Fig. 4 that direct-far field measurements for such a transducer require a very large tank ($\mu \gtrsim 120$ wavelengths per tank side) or absorption so large that the tank is essentially anechoic. On the other hand, measurements in the direct-near field appear to require a moderately large ($\mu \gtrsim 30$) or anechoic tank if

we are to avoid the risk of significant room reaction. (This last conclusion is very tentative because of lack of knowledge about the near field.)

Figure 5 presents similar data for a transducer with maximum directivity factor $D_o = 30$ (directivity index = 15 db). Because of the greater directivity the direct-reverberant boundary moves to larger distances r (or lower frequencies or smaller tanks). Our scheme for estimating the probability of significant room reaction is not dependent on D_o, so that no change is seen in that boundary. The general conclusions in this case are similar to those given above for Fig. 4, except that direct field measurements can be made at lower frequencies. However we note the probability of difficulties in this case (particularly with $\bar{a} = 0.25$) arising from the direct field dominating the reverberant field on some areas of the wall [Eq. (33) and Table I].

IV. MEASUREMENT TECHNIQUES IN TANKS

In this chapter we survey the various possible techniques for abstracting information about the performance characteristics of a transducer by measurements of the sound field it produces in a water-filled tank. Insofar as possible, we determine the limitations on the sort of information which can be gathered by each technique.

A. MEASUREMENTS IN REVERBERANT FIELD

Measurements of the steady-state pure-tone sound at a point where the reverberant field predominates can reveal no information about a transducer except its power output at the frequency of the signal. In particular, the directivity pattern (the direct field) cannot be ascertained. Measurements with a noise signal may reveal some more information.

1. Pachner's Method

Pachner (1956b) proposed an elaborate method for determining the directivity pattern which involved measuring instantaneous pressure amplitudes at a large number of positions on each of two spheres surrounding the source. The spheres are chosen conveniently to differ in radius by $\lambda/4$ (approximately). Measurements are made at each position at each of two phases in the cycle, differing by $\pi/4$ radians.

We will only note in passing the experimental difficulty of achieving sufficient precision in the measurements to maintain accuracy in the prediction of the direct field. (The direct field pressures at the measurements points are only a small part of the total pressure measured.)

A more fundamental difficulty with Pachner's method lies in his theory. He appears to assume that the reverberant field (defined as total field less direct or "free" field) is a pure standing wave, carrying no power. As noted in the earlier discussion ("Direct and reverberant fields"), this concept is false, as can be shown by simple physical considerations: The flow of power at the walls is controlled by the distribution of absorption on the walls. Analytical demonstration of this fact can be found from the results in Appendix I, where it is shown that the real part of the Poynting vector (i.e., the intensity vector which describes the flow of power) satisfies Laplace's equation in the tank [Appendix I, Eq. (8)]. The boundary condition at the wall (in the reverberant field) is determined by the reverberant field pressure.

Although Pachner's method cannot yield correct information about the directivity pattern, it may, in principle - that is, with perfectly precise measurements, be used to predict total power radiated. In this connection, two warnings must be sounded concerning details of Pachner's paper.

First, Pachner employs a well-known result based on the superposition principle: A given field can be described alternatively as (1) the sum of two waves, one travelling towards and one away from a surface, or (2) the sum of one standing wave and one residual travelling wave. However the superposition principle is not applicable to sound power, which is a squared quantity. The power flow through a surface equals the difference between the powers carried by the two travelling waves, using the first description of the total field. This power is <u>not</u> equal to the power carried by the <u>residual</u> travelling wave, using the second description of the total field.

The second warning concerns an error of detail made by Pachner between his Equations (1) and (1a) which restricts the generality of the remainder of his paper. He has apparently assumed that the coefficients, α, β, γ, δ in his Eq. (1) are real. Since a similar assumption and similar restrictions are implicit also in Pachner's free-field method (1956a), we defer further comment to our discussion below of that simpler case.

2. Intensity Method

It is attractive to consider the possibility of determining the direct field directivity by direct measurements of intensity in the reverberant field. However this technique is subject to the same criticisms as to both experimental accuracy and theoretical basis as is Pachner's method. The intensity method can, in principle, be used to determine the total power radiated, but not the directivity pattern. This method is discussed in greater detail in Appendix II.

3. Correlation Method

Stroh (1959) has used correlation techniques to determine the direct field of a noise source in a room. However the method is not applicable to a pure-tone signal as he has pointed out in an unpublished communication (Stroh, 1961); his comments are summarized here.

In order to measure the energy density of the direct field in the presence of reverberation, it is required that the cross-correlation, with zero delay, of direct and reverberant fields be negligibly small [the term P in Eq. (9) of Stroh, 1959]. Suppose the direct signal is narrow-band noise with a fractional

bandwidth $\Delta f/f \equiv 1/Q$. Let the reverberant field be described as the field of a set of images. If the bandwidth is great enough, the cross-correlation between direct and reverberant fields will resemble the sketch in Fig. 6: a series of "bumps" spaced along the τ-axis at delays corresponding to the travel time from each image to the observation point. Each bump has more or less the same shape as the autocorrelation function of the direct signal. (Differences will arise from the dependence of directivity pattern upon frequency.)

If the amplitude of cross-correlation at delay $\tau = 0$ is to be small, the breadth $\Delta\tau$ of each bump must be small:

$$\Delta\tau \ll r_1/c \quad ,$$

where r_1 is the difference between the distances to the observation point from source and from nearest image. Now, the breadth of an autocorrelation function is related to the bandwidth of the signal (in frequency) by a sort of uncertainty relation:

$$\Delta f \, \Delta\tau \approx 1 \quad .$$

Therefore the requirement can be rewritten

$$1/\Delta f \ll r_1/c \quad , \text{ or } Q \ll r_1/\lambda \quad ,$$

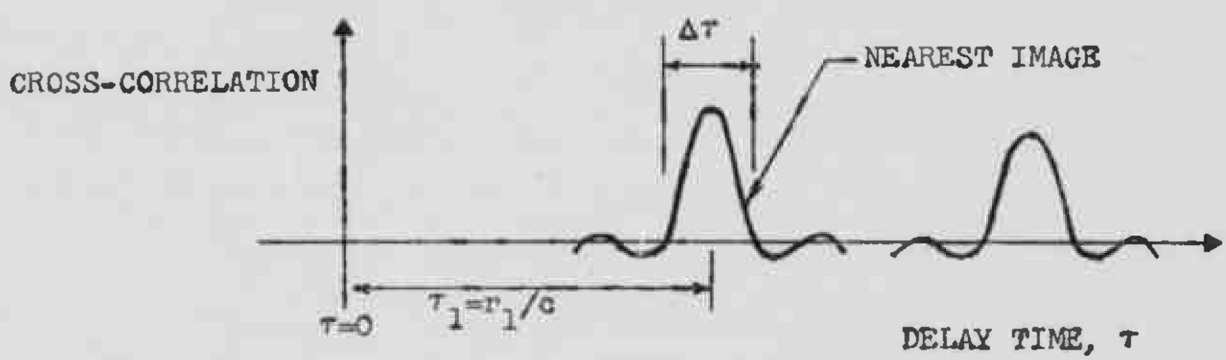


FIG. 6 SCHEMATIC DRAWING OF CROSS-CORRELATION BETWEEN DIRECT AND IMAGE SIGNALS

where λ is wavelength at the center frequency. The maximum value of r_1 is, roughly speaking, the characteristic length $L_c = V^{1/3}$ of the tank. Therefore the criterion for usefulness of the correlation method in detecting the direct signal in the presence of the reverberation is

$$\mu \gg Q, \text{ where } \mu \equiv V^{1/3}/\lambda \quad . \tag{36}$$

We have considered a noise signal with fractional bandwidth 1/Q. A pure tone has zero bandwidth and infinite Q. Therefore the method does not work with a pure tone.

The limitation, Eq. (36), will often be satisfied if the transducer is driven by a broad band of noise so that the bandwidth (1/Q) is set by the transducer itself. (Typical values of Q range from 4 to 30.) However such measurements can only reveal average characteristics of the transducer--efficiency, directivity, etc., averaged over frequency. Whether such limited information is sometimes adequate for transducer evaluation and calibration is a question that should be studied.

In a large tank, some more detailed information could be obtained by exciting the transducer with a noise signal whose bandwidth is narrower than the transducer's and whose center frequency is variable. For example, if the transducer's Q is 4, the noise bandwidth could be chosen to have Q = 10. Correlation measurements in the reverberant field of a tank with $\mu \approx 30$ (i.e., 30 wavelengths on a side) might yield reasonably accurate results.

There is one potentially serious problem that arises in connection with correlation measurements with large transducers comprising many resonant elements: How does one derive the "direct signal" which is to be electronically correlated with the direct-plus-reverberant signal? If the bandwidth of the electrical noise input is larger than the bandwidth of the resonant elements, the spectra of outputs of various elements may differ. It appears that no unique "direct signal" can be defined in this case, and that the usefulness of the correlation technique may be degraded. On the other hand, the usefulness should be preserved if the bandwidth of the electrical signal can be made narrower than the bandwidth of the elements.

4. Pulse Method

Pulse techniques are standard for the calibration of transducers in tanks. By them one can sometimes measure direct field pressures at a point in the tank where the reverberant field would predominate in the steady state. The transducer is activated by a pulse-modulated carrier; under certain conditions, the response will have reached essentially its steady state value and will then have propagated directly to the observation point before any energy reaches that point via a reflection from the walls.

The conditions for successful use of the pulse method are identical with the conditions for successful use of the correlation method. If a resonant system is pulsed, the duration $\Delta\tau$ of the initial transient is generally related to the bandwidth Δf of the system by an uncertainty relation:

$$\Delta f \, \Delta\tau \approx 1 \quad .$$

The condition for success is that $\Delta\tau$ shall be small compared with the difference in travel times from source to observation point via direct and reflected paths. In the image view, this difference is just the travel time to the observation point from the nearest image. Thus the analytical expression of the criterion is

$$\Delta\tau \ll r_1/c \quad ,$$

where r_1 is the difference between the distances to the observation point from source and from nearest image.

This criterion and the uncertainty relation are analytically identical to the corresponding expressions exhibited above in the analyses of the correlation method. Therefore the combined criterion for usefulness [Eq. (36)]

$$\mu \gg Q \tag{36}$$

where $\mu = V^{1/3}/\lambda$

$Q = f/\Delta f$,

is equally applicable to pulse and correlation methods. It appears that, at least for a linear transducer, the two methods are equivalent; in a given tank, each should be capable of yielding answers with the same precision.

5. Total Power Measurements

The determination of total sound power output of a source by measuring the pressure in a region where the reverberant field predominates is a standard method used in architectural acoustics. However, it must be recognized that the method is explicitly not recommended for use with narrow-band or pure-tone signals, in recent technical summaries (American Standards Association, 1960).

The reasons for this restriction were two-fold. First, it may be difficult to determine a reliable average sound pressure for the room. Secondly, the variation from point to point of the radiation impedance may be sufficient to make the source position critical.

Our estimates of the radiation impedance problem which were presented above (Chapter III, section 2) indicated that it should be of considerable concern unless the tank is quite large. We also indicated the need for much more study of the problem. (See Chapter II, "Room reaction on the source.")

Another difficulty arises with very directive sources. Unless the reverberant field dominates the direct field on all tank walls, the power-pressure relations will depend on the source position and orientation; the tank cannot be calibrated by means of a different, standard source. However predominance of the reverberant field at the wall was seen to require very small absorption, increasingly so with increasing directivity. (See Chapter III, section 3, above.) This small absorption will, on the other hand, put more stringent requirements on tank size if the radiation impedance problem is to be avoided.

B. MEASUREMENTS IN DIRECT FIELD

By measurements in a region where the direct field predominates, information can be gathered about both the directivity pattern and the total power radiated.

However, as a practical matter, the sound in such a region will be polluted to some extent by reverberant energy, unless the transducer is in the open sea. This pollution is more severe with large transducers because the direct field intensity is less than that of a small source emitting the same total power. The effect of this residual reverberant sound upon the accuracy of measurements by any proposed scheme needs to be assessed; in most cases the effect has not been adequately discussed.

1. Pressure Method (Far Field Only)

The classical example of measurements made in the direct field is the determination of source power and directivity by measuring pressure in the far field. Such measurements require very large tanks, even if one's interest is restricted to the main lobe of radiation, unless the tank can be made anechoic. (See Figs. 4, 5.). Even an anechoic tank cannot be small, if the transducer is large, because the near field extends to such a distance. (See Fig. 3.) For example, the near field of a transducer whose maximum dimension is about 4λ extends to a range of 12λ.

2. Horton's Method

Horton has investigated a method of predicting far-field pressures from measurements made near the transducer, which is based on the Helmholtz integral relation between the pressure at a distance point and the pressure and velocity at all points on a closed surface surrounding the source (Horton and Innis, 1960, 1961).

The method is based on an analysis that assumes the transducer is the only source in an infinite, "free" space. The question of pollution by reverberant energy has not been considered. Therefore, the utility of the method for tank methods cannot be assessed.

There does appear to be some discrimination against the reverberant field inherent in Horton's method when it is used on a large transducer. The reverberant field pressures and velocities on the measurement surface act as additional source terms in the Helmholtz integral. On a large surface, the phase (and direction and amplitude) of these pressures and velocities will fluctuate from area to area. There is therefore a tendency to cancellation in the net effect due to these terms. However the same fluctuations could introduce disturbing perturbations in those details of the predicted directivity pattern which depend intimately on phase differences between different points of the source.

In his "realistic approximation," Horton makes do with measurements of pressure alone, computing particle velocity on the assumption that

$$p \approx v\rho c \quad .$$

This assumption is known to be accurate in the far field or at points near the transducer's surface if (1) the point is not near the transducer's edge, and (2) the transducer is fairly large ($r > \lambda$). This assumption has been reported to have been, in some cases, the cause of inaccuracies in Horton's predictions. Availability of a good velocity (pressure gradient) transducer would obviate the necessity for the assumption.

3. Pachner's Method

Pachner (1956a) proposed a method for predicting the far field of a transducer in free space from measurements of the near field on a surface surrounding the transducer. This method is somewhat simpler than that he proposed for use in a reverberant tank (Pachner 1956b). Horton and Innis (1961) have shown that Pachner's free-field method is equivalent to Horton's method, but less general in that the surface on which measurements are made cannot be chosen with complete freedom. The surface must be one for which a complete set of orthogonal harmonic functions are known. (Pachner chose a sphere as measurement surface; Horton and Innis generalized Pachner's method to apply to a prolate spheroid.)

Pachner's method involves the measurement of instantaneous pressure amplitudes at each of a large number of points on the measurement surface enclosing the transducer; measurements are made at each of two phases of the cycle, conveniently chosen to differ by $\pi/2$ (i.e., one-quarter period in time). By linear combinations of these data, the complex modal amplitudes of a modal expansion of the field on the measurement surface are determined. By restricting the choice of surface, for example to spheres, prediction of the far field from this near-field modal expansion is made relatively simple.

In essence, Pachner proposes to describe the actual transducer by an assortment of multi-pole sources (monopole, dipole, etc.) located at the center of his measurement sphere. His "method" is a procedure for determining the relative strengths of these fictitious multi-pole sources from instantaneous pressure measurements on the sphere.

In principle, this scheme appears perfectly correct and general. Unfortunately Pachner has made implicit assumptions that restrict the generality of his calculation procedure. It is readily seen, from a comparison of Pachner's Equations (1) and (1a) that Pachner implicitly assumes that the coefficients a_{mn} and b_{mn} are real. [See also his Eqs. (8) and (9) which express these coefficients in terms of real numbers.]

The physical significance of this assumption is a restriction on the generality of the source. Let us build up a general source from an assortment of multi-poles. First take a monopole (simple source); we must specify both its strength and time phase. Secondly, add a dipole; we must specify <u>three</u> quantities: its strength, its orientation, its time phase. But Pachner has only two coefficients, a_{mn} and b_{mn}. By assuming both of them are real, he is implicitly restricting the time phase. Similar restrictions apply to all sources of higher order.

It appears, therefore, that Pachner's procedure is not valid for the general transducer.* However this discussion does not reveal the conditions under which it may be valid.

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* These remarks do not reflect upon the validity of Horton's method, despite the demonstration by Horton and Innis (1961) of an "equivalence" between Pachner's and Horton's methods. That demonstration can be interpreted as either (1) a demonstration of equivalence <u>in principle</u>, with complex modal coefficients (i.e., without Pachner's implicit restriction), or (2) a demonstration for the special case where the source surface is a separable coordinate surface (spheroid) and all points on it move with the same time-phase, in which case Pachner's method is valid.

Were Pachner's free-field method (1956a) used for measurements in a tank with the measurement surface located where the direct field predominates, the question of errors due to residual reverberant field must be raised. The comments on this point in the immediately preceding section (Horton's method) are equally pertinent here. We note however that the greater freedom of choice of measurement surface which is inherent in Horton's method allows one to locate the surface closer tc the transducer, where the predominance of direct field should be greater.

Pachner's second method (1956b) could be used in an attempt to discriminate against pollution by the residual reverberant field. We indicated in the discussion above that the theoretical basis of that second method is faulty. However, such a statement must not be taken to mean that the more complex method is completely useless. Pachner's hope of complete discrimination against the reverberant field is an idle one, yet some discrimination could result, and this little could be sufficient. The question needs careful investigation. At the same time one should inquire whether the potential improvement is greater than the degradation which would result from the increased number of measurements, each with its inevitable inaccuracies. (In calculations involving lengthy arithmetic manipulations of numerous data, the propagation of errors becomes a very serious problem.)

4. Intensity Method

Given a perfect intensity meter and a transducer in an infinite medium, one can, presumably, make measurements over any surface enclosing the transducer and predict therefrom the far field. It appears that this intensity method has only been proposed conjecturally, and has not been the subject of serious study.

A basis for further consideration of the intensity method is outlined in Appendix I. In an ideal medium without sources or sinks, such as the medium external to the measurement surface in the hypothetical experiment above, the time-averaged intensity vector satisfies Laplace's equation [Appendix I, Eq. (8)]. Procedures for calculating the far field therefore correspond to solving what appears to be a reasonably simple, static boundary-value problem.

Note that analysis will be necessary in order to predict far-field directivity patterns from near-field measurements. Static fields, e.g., electrostatic, are subject to near-field distortions equally as much as are dynamic fields. However, it appears that the calculations may be simpler in the static case (intensity method) than in the dynamic (Horton's method).

Now consider intensity measurements made in a tank at points where the direct field predominates. The same ugly question of "pollution" by the reverberant field must be raised again. Again, the question requires study. (Of course the total power output will be properly assessed in any case; but the reverberant field may distort the calculated directivity pattern.) A more detailed discussion of intensity measurements is given in Appendix II.

V. PROCEDURES FOR MODIFYING TANK ABSORPTION

Throughout the previous discussion of tank acoustics, the average sound absorptivity of the walls has played a critical role. It is evident that the ability to tailor the absorptivity to a desired value may be essential to the construction of a successful calibration tank.

In this chapter we summarize the current information about absorptivity and means for modifying it. The efficiency of metallic plates treated with mechanical damping material is given special attention.

A. INHERENT TANK ABSORPTION

A natural first question is: How much absorption has an untreated tank? There is little data available at the present time, but these indicate that absorption can be quite high.

Klein (1960b) mentions some reverberation experiments at 2kc/s in one glass tank (3/4-in. glass in a steel frame, 1200 ft^3) and one steel tank (plates in a heavy frame, 1440 ft^3). The natural reverberation times were so short that the effect of artificial sound absorbers could hardly be detected.

Klein (1961) reports some other data furnished by R. W. Young (Navy Electronics Laboratory) and V. Salmon (Stanford Research Institute). Young measured a reverberation time of about 0.15 sec in a rectangular concrete tank whose description follows:

30 ft x 12 ft x 7.5 ft deep
water depth: 5 ft
walls: 8 in. of reinforced concrete
bottom: 4 in. of reinforced concrete, set on ground.

The calculated Sabine absorption is therefore about 140 sabins (ft^2). If this value is divided by the wetted area of concrete (780 ft^2), the resulting value for average Sabine coefficient is $\bar{a} \approx 0.2$. If one divides by the area of the bottom, one obtains $\bar{a} \approx 0.35$.*

Salmon's data (which Salmon carefully labels "very preliminary") were taken in a cylindrical steel tank:

8 ft diameter x 12 ft high
water depth: 11 ft
walls: c. 3/16-in. steel, backed by air
bottom: steel on wood pad on concrete.

The natural reverberation time, measured by a tone warbled around 5 kc/s, was about 0.45 sec. The Sabine absorption is therefore about 14 sabins. The average Sabine coefficient, calculated from total wetted wall area, is $\bar{a} \approx 0.04$; but, calculated from bottom area only, $\bar{a} \approx 0.3$.

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* In the earlier part of this report we consistently used an estimated area $S = 6V^{2/3}$. In a tank more nearly cubical than the present one, $6V^{2/3}$ is a good estimate of the total surface area <u>including</u> the top surface of the water. In the present case, $6V^{2/3}$ is but 10% less than the area of wetted concrete.

These few data make one suspect that many tanks, particularly those of lighter construction, inherently have a very non-uniform distribution of reflectivity, and that sound energy of these relatively low frequencies passes quite easily through walls which are in intimate mechanical contact with the ground. Since most vertical walls are backed by air, and the top water surface is perfectly reflective, a very "non-random" distribution of absorption may be characteristic of existing tanks. If such be the case a fundamental assumption of all "high-frequency" theories of room acoustics - namely, diffuseness - is open to serious question.

These data also indicate that reflective treatments of some walls may be necessary before a tank can be used for reverberant testing. Unfortunately, natural absorption is not adequate for quasi-anechoic testing.

A survey of the walls of various tanks with an intensity meter would reveal some most interesting and valuable data about their absorptive characteristics.

B. ABSORPTIVE TREATMENTS

There has been much experimentation with absorptive wall treatments for water-filled tanks. In general, the treatments most successful at lower frequencies have employed wedges made of lossy rubbery materials having inclusions of air. Cramer (1960) presents an excellent survey of existing anechoic structures. Thorough summaries of unclassified information and theory about broadband and resonant absorbers have been given by Tamm (1957) and Oberst (1957).

Recent reports on specific treatments have been made by Cramer and Johnston (1956), Toulis (1955), and Meyer et al. (1960). The last two describe treatments for which the amplitude reflection coefficient is less than about 0.1 at frequencies above 5 kc/s but increases quite markedly below that frequency.

Successful anechoic treatments for frequencies below 5 kc/s appear not to have been developed to date. One might speculate that designing for lower frequencies may involve much more than a simple scaling of the treatment. Because it is unlikely that the walls of a tank will be suitably scaled (in thickness as well as length), it is likely that the flexibility of the walls that back up the treatment will become increasingly important at lower frequencies.

We present in the next chapter an analysis of the potential for sound absorption of one particular structure: metal plates treated with mechanical damping layers.

VI. THE DAMPED PLATE AS A SOUND ABSORBER

We present here a preliminary analysis of the sound absorption possibilities inherent in metal plates which have been treated for maximum mechanical damping.

The flexural vibration of metal plates can be quite highly damped by interleaving them with layers of viscoelastic materials. Procedures for designing such multilayered damped structures have been quite highly developed (Ross, Ungar, and Kerwin, 1959). No one would suggest the use of these structures as sound absorbers in <u>air</u>, but their potentialities in <u>water</u>, which has a much larger characteristic impedance, are intriguing.

Consider an elastic plate which executes bending vibrations under the action of sound. We assume that the plate is damped by virtue of layers of viscoelastic and elastic materials applied to it. The damping is characterized by the <u>loss factor</u> η, which is related to the reciprocal fractional bandwidth, Q, and the (amplitude) decay rate K by

$$\eta = 1/Q = K/\pi f \quad ,$$

where f is the frequency, if η is small. (The quantity $\pi\eta$ is also called the "logarithmic decrement".)

1. Infinite Plate Analysis

Consider first the special case of an infinite flat plate exposed to sound on one side only; we assume that the incident sound is a plane wave. The results should be quite accurate for any large damped plate in which the separation of the eigenfrequencies is not larger than the bandwidth of individual eigenmodes, in the frequency band of interest. We expect the resulting estimate of sound absorption to be optimistic (i.e., too large) because of the assumption of a plane incident sound wave; the coupling to a spherical wave, for example, should be less. The assumption that only one side of the plate is exposed to sound also seems to be optimistic. The effective force is proportional to the pressure difference between the two sides. If sound acts on both sides, the pressure difference may be much reduced, or it may be increased by as much as a factor of 2, relative to exposure of one side only. However, the reradiated sound, which sets an upper limit to the absorptivity, will be doubled by exposure of both sides.

The first step in analyzing the special case is to determine the amplitude of response to a specified incident plane wave. The forced response of the plate will be small except when perfect "trace-matching" occurs between the incident sound wave and the free elastic wave with the same frequency. "Trace-matching" refers to the condition that the tangential phase velocity of the sound wave equal the free-wave velocity c_b for the plate. If the sound wave is incident at an angle θ measured from the normal, the trace-matching condition is

$$c = c_b \cos\theta \quad . \tag{37}$$

Can trace-matching be achieved? The answer seems to be: No, not with bending waves in reasonably homogeneous flat plates. The speed of bending waves in such plates is too small. In a flat plate, the speed of bending waves can be written (Junger, 1960 b, Eq. II.5)

$$c_b = c_L \, (2\pi \, r_g/\lambda) \tag{38}$$

where

c_L is the velocity of a dilational elastic wave,

r_g is the radius of gyration of the section,

λ is the wavelength.

In a uniform plate

$$2\pi \, r_g = 1.8 \, h$$

where h is the total thickness. In common materials the speed c_L is not greater than about

$$c_L \lesssim 1.8 \times 10^4 \text{ ft/sec} \quad ,$$

the value for steel. A fundamental condition for a bending wave is that the wavelength be large compared with the thickness, say:

$$h/\lambda \lesssim 0.1 \quad .$$

Therefore the bending wave speed cannot exceed

$$c_b \lesssim 3 \times 10^3 \text{ ft/sec}$$

and is much too small for trace-matching to water waves ($c = 5 \times 10^3$ ft/sec) at any angle of incidence ($\cos\theta \leq 1$). [See Eq. (37)].

We therefore conclude that trace-matching to simple bending waves cannot be achieved. Trace-matching might be achieved between the wave in the water and some elastic wave which is more or less restricted to the surface of the plate, but in that case it is quite likely that the damping will be reduced, for the following reason. The loss factor η is a measure of the ratio of dissipated energy to stored energy. The greater elastic wave speed required for trace-matching can be achieved only by an effective increase in the stiffness of the structure and thus an increase in the stored energy and a decrease in η.

It would be possible, by means of a standard scattering analysis, to show that

1) even if trace-matching can be achieved, the energy absorption coefficient of the plate cannot exceed 0.5, corresponding to half the incident energy being reflected and half being absorbed; and

2) that the value 0.5 cannot be achieved with reasonable damping treatments ($\eta \approx 0.1$) if one assumes $h/\lambda \leq 0.1$.

However, these results would be somewhat limited by the assumption that the plate is infinite. Therefore we sketch an analysis based on a modal description of the plate's motion which is somewhat more general and is certainly more pertinent to not too large plates which are mildly resonant.

2. Modal Analysis

The absorbing structure vibrates in response to an incident plane sound wave characterized by the free-field pressure p_o. The amplitude of response is characterized by a reference velocity $v = v_o \exp(j\omega t)$. The dynamics of modal response are expressed in modal impedances defined from the energies of motion. Thus the Lagrange equation for the single mode is

$$F = v\,(R_m + jX_m) \quad , \tag{39}$$

where the mechanical power dissipated is

$$\Pi_m = \frac{1}{2}\, v_o^2\, R_m \quad . \tag{40}$$

The generalized force F includes two components:

a) that due to the incident wave (the generalized modal force that would be evaluated from the sound pressure on the structure when it does not move), and

b) that due to radiation reaction. Thus

$$F = F_o - F_{rad} \quad . \tag{41}$$

The radiation reaction includes both resistance and reactance:

$$F_{rad} = v\,(R_{rad} + jX_{rad}) \quad , \tag{42}$$

and the power reradiated from the vibrating structure is

$$\Pi_{rad} = \frac{1}{2}\, v_o^2\, R_{rad} \quad . \tag{43}$$

Now the generalized force F_o can be related to the radiation resistance R_{rad} with considerable generality by reciprocity considerations (Smith 1960). This relationship is similar to the reciprocity between (1) the efficiency with which a plane wave, incident from a given direction, excites motion, and (2) the efficiency with which that motion radiates back in the same direction. The relationship desired here is*

$$F_o^2 = 4\pi\, p_o^2\, D\, R_{rad}/\rho c k^2 \tag{44}$$

where

D is the directivity factor, in the incident direction, of the structure as a radiator;

$k = \omega/c = 2\pi/\lambda$;

p_o is the amplitude of incident sound pressure.

The combination of the preceding equations yields

$$\pi_m = \left(\frac{p_o^2}{2\rho c}\right) \frac{4R_m/R_{rad}}{(1+R_m/R_{rad})^2}\, \frac{\lambda^2 D}{4\pi}\, \frac{1}{1+\left(\dfrac{X_m + X_{rad}}{R_m + R_{rad}}\right)^2} \quad . \tag{45}$$

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* The reference cited (Smith 1960) expressed the radiation resistance in terms of the space-average value of the square of a transfer function, and did not, unfortunately, introduce the directivity function in the form of Eq. (44). However, the present result follows directly from the standard definition of D and Eqs. (12) and (15) of the reference.

The first term on the right is the intensity of the incident sound wave. The last term is unity at resonance ($X_m + X_{rad} = 0$) and shows the degradation in absorption if the structure is not operating at resonance. The second term on the right has a maximum value of unity when

$$R_m = R_{rad} \text{ for maximum absorption .} \tag{46}$$

Thus the most power that can be dissipated in the absorbing structure is equal to the power that is reradiated. (Stated in another jargon, we conclude that the ratio of absorption to scattering cross sections never exceeds unity.)

Assume the structure can be and is designed for resonance and for $R_m = R_{rad}$. Then this maximum energy absorptivity of the structure is

$$a_{max} \equiv \Pi_m/(p_o^2/2\rho c) = \lambda^2 D/4\pi \quad . \tag{47}$$

If the structure is large and highly directive, the maximum directivity factor has been estimated as (Stenzel, 1958; "general proposition" at end of Part I, p. 53)

$$D \approx 2\pi S_o/\lambda^2 \quad , \tag{48}$$

where S_o is the surface area of the structure. Thus

$$a_{max} \approx 0.5\ S_o \text{ (very directive case)} \quad , \tag{49}$$

corresponding to an energy absorption coefficient of 0.5.

If the structure is small and non-directive ($D = 1$), the absorptivity becomes increasingly large as frequency decreases and λ increases. However, that result is deceptive since it pertains to a single absorbing structure. If more than one absorber is used, they must be widely spaced if the analysis is to be valid. The spacing required is not revealed by this analysis, but it must be some constant number of wavelengths, in which case the area per absorber is a constant number of λ^2. This sort of variation just cancels the apparent advantage to be found by using large λ; the absorption coefficient (per unit of wall area) will not increase. (One may, with advantage, consider the array of absorbers as a combined structure with large area and high directivity.) Some of the design problems inherent in small absorbing structures have been discussed at length by Ingard (1953).

Let us return to the general conditions for maximum absorptivity. There are three conditions which must be satisfied simultaneously.

1) resonance ($X_m + X_{rad} = 0$);

2) high directivity ($\lambda^2 D$ as large as possible);

3) resistance balance ($R_m = R_{rad}$).

There is no difficulty satisfying the first condition by itself. There may be great difficulty satisfying the first two conditions at the same time. An example of that difficulty is found in the earlier discussion of absorption by an infinite plate where we concluded that trace-matching with a simple bending wave is impossible. When the free flexural wave (i.e., resonant motion) has a speed lower than the speed of sound in the fluid (i.e.,

trace-matching is impossible), high directivity cannot be achieved. [On this point, see Junger's short discussion (1960b, Section V) and the literature to which he refers.]

However, suppose the first two conditions were, somehow, satisfied. Probably this requires a fairly large structure on which the ratio of sound pressure radiated to normal velocity will approximately equal ρc. (See Lax and Feshbach, 1947.) Then the resistance balance will require

$$\frac{R_m}{R_{rad}} \approx \frac{\eta\omega m}{\rho c} \approx 1$$

where m is the mass per unit area of the structure. Replace m with the product of density ρ_p and thickness h;

$$\frac{R_m}{R_{rad}} \approx 2\pi\eta \frac{\rho_p}{\rho} \frac{h}{\lambda} \quad .$$

For bending waves, we require, at least,

$$h/\lambda \lesssim 0.1 \quad .$$

With a steel structure, we have

$$\rho_p/\rho \approx 8 \quad .$$

Therefore the resistance balance requires a loss factor

$$\eta \gtrsim 0.5 \quad ,$$

which seems too large for any practical design of multilayered damped plate (Ross, Ungar, and Kerwin, 1959). A more readily achievable loss factor, $\eta = 0.1$, results in about a 50% reduction in the absorption. [See Eq. (45).]

3. Summary

This survey of the possibilities for achieving high sound absorption with a damped mechanical structure indicates three general conditions (resonance, high directivity factor for the direction of incidence, and resistance balance) which must be met simultaneously. However, no means for satisfying the three conditions with a damped plate, vibrating in bending, are apparent.

VII. AREAS REQUIRING FURTHER STUDY

The results of this survey of water-tank acoustics suggest the necessity for study of a number of topics. Throughout this survey of water-tank acoustics we have been facing a dilemma: Make outright guesses, and believe the answer if you dare; or find only qualitative answers of little engineering utility. This reflection of our ignorance is not a happy circumstance.

It is clearly evident that a number of topics require much more study if our armament of basic understanding, predictive ability, and experimental facility is to be adequate for solving the problems that will arise at low frequencies. We describe these topics briefly in this chapter, recognizing that some are already being studied.

1. Test Specifications for Production and Maintenance

As transducers grow larger and frequencies turn lower, one must abandon the old test apparatus. The old test procedures should be reviewed critically at the same time. It is quite possible that new procedures can be devised which will be adequate and, at the same time, cheaper and more efficient than a scaled-up version of the old. Perhaps production and maintenance tests on transducers do not require a tank experiment. Now is the time to find out, before new and larger facilities have been built on the old model.

2. Absorptive Treatments for Low Frequencies

None of the absorptive treatments which have been developed for making tanks anechoic has been proved effective at frequencies below 5 kc/s. Experimental research, adapting treatments proven at higher frequencies, should be fruitful. However, new techniques may be necessary for frequencies much lower than 5 kc/s.

3. Practical Application of Intensity Meters

Many intensity meters have been built; many potential applications have been described (see Appendix II); no routine use of an intensity meter has been made. The principal requirements at this time are (1) detailed experimental proof that the meters have practical value, and (2) greater practical experience with the devices.

4. Near Field of Transducers

Present knowledge of the near field of large and complicated transducers generally falls into one of two classes; collations of data from individual cases, and elaborations of mathematical procedures. Simplified means for predicting various characteristics of the near field of a practical transducer are urgently required, whether these be analytical approximations of general applicability or experimental "rules of thumb." At the present time, even prediction of the location of the "boundary of the near field" must be equivocal. The energetic descriptions of sound fields outlined in Appendix I may be a useful analytical tool for forming a new view of the general problem.

5. Near-Field Measurement Schemes

Horton's method for predicting far-field characteristics from near-field measurements is currently being studied and tested. However, this work is under some restriction for lack of an adequate pressure-gradient meter. Neither Pachner's nor the intensity method is being considered. None of the schemes has been used in tanks.

If any of these methods is to be used in an enclosed space, the question of errors due to residual reverberant field should be studied in detail. In connection with the intensity method, the possibilities of processing data on an electrostatic analog are very attractive.

6. Tank Acoustics

A number of aspects of tank acoustics require further study in an attempt to improve our ability to predict sound fields.*

Some topics are:

a) Reaction on the source, changes in radiation impedances due to reverberant field;

b) Tank response to narrow-band noise (most analyses are limited to pure tones or broad-band noise);

c) Energy relations (energetic descriptions of sound fields have not generally been exploited; they furnish a different viewpoint which may increase our understanding and knowledge).

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* Note that these items should also interest the Navy for their value in "air-tank" (room) acoustics. The same problems arise in connection with measurement of the sound output of machinery which is to be installed in naval vessels. There, too, tests must be made under conditions not ideal from the viewpoint of a standards laboratory.

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APPENDIX I

PRELIMINARY INVESTIGATION OF ENERGY RELATIONS FOR A STEADY SOUND FIELD

We summarize here, with sketches of derivations, some useful relationships for energy functions in a small-amplitude sound field in a perfect fluid characterized by density ρ and sound speed c. Some of the results are implicit or explicit in a number of analyses in the literature.[1]

We use complex notation for the variables, with the time dependence $\exp(j\omega t)$. Alternatively the variables in this analyses may be considered to be the Laplace (or complex Fourier) transforms of the real, instantaneous variables of the sound field. We define, with vectors indicated by underscoring:

$\underline{r}$	position vector,
t	time variable,
$p(\underline{r},t)$	sound pressure,
$\underline{v}(\underline{r},t)$	particle velocity vector,
$\underline{d}(\underline{r},t)$	particle displacement vector, $\underline{v} = \dot{\underline{d}} = j\omega\underline{d}$.

The time-averaged energy densities are:[1,2]

time-averaged kinetic energy density, $T = \frac{1}{4}\rho\underline{v}\cdot\underline{v}^*$,

time-averaged potential energy density, $V = \frac{1}{4}p\cdot p^*/\rho c^2$, (1)

time-averaged Lagrangian density, $L \equiv T-V$,

where the superscript star indicates a complex conjugate.

We further define the complex Poynting vector

$$\underline{J} = \underline{P} + j\underline{Q} \equiv \tfrac{1}{2}p\underline{v}^* \tag{2}$$

which is, of course, independent of time. The vectors $\underline{P}$ and $\underline{Q}$ are the real and reactive intensities.

Away from sources and sinks, the sound field in a perfect fluid must satisfy the force equation[3/]

$$\underline{\nabla}p = -\rho\dot{\underline{v}} = -j\omega\rho\underline{v} \quad , \tag{3}$$

and the combination of the equation of continuity and equation of state:[3/]

$$p = -\rho c^2\underline{\nabla}\cdot\underline{d} = j(\rho c^2/\omega)\underline{\nabla}\cdot\underline{v} \quad . \tag{4}$$

Now we compute, by standard formulae of vector analyses, from Eqs. (1)-(4)

$$\begin{aligned} 2\,\underline{\nabla}\cdot\underline{J} &= \underline{\nabla}\cdot(p\underline{v}^*) = \underline{v}^*\cdot\underline{\nabla}p + p\nabla.\underline{v}^* \\ &= -j\omega\rho\underline{v}\cdot\underline{v}^* + j\omega p\cdot p^*/\rho c^2, \text{ or} \\ \underline{\nabla}\cdot\underline{J} &= -j2\omega(T-V) = -j2\omega L \quad . \end{aligned} \tag{5}$$

Since the Lagrangian density L is by definition real, the real and imaginary parts of this equation can be written

$$\begin{aligned} \nabla\cdot\underline{P} &= 0 \\ \nabla\cdot\underline{Q} &= -2\omega L \quad . \end{aligned} \tag{6}$$

Now consider the curl of the Poynting vector $\underline{J}$ in an irrotational sound field where the curl o. particle velocity vanishes. We compute, with the help of Eq. (3),

$$2\,\underline{\nabla}x\underline{J} = \underline{\nabla}xp\underline{v}^* = p\underline{\nabla}x\underline{v}^* + \underline{\nabla}px\underline{v}^*$$
$$= \underline{\nabla}px\underline{v}^* = -j\omega\rho\underline{v}x\underline{v}^* = 0$$

since $\underline{v}$ and $\underline{v}^*$ are parallel vectors. Thus

$$\underline{\nabla}x\underline{J} = \underline{\nabla}x\underline{P} = \underline{\nabla}x\underline{Q} = 0 \ . \tag{7}$$

It is evident from Eqs. (6) and (7) that $\underline{P}$ satisfies the vector Laplace equation

$$\nabla^2\underline{P} = 0 \tag{8}$$

everywhere in the fluid, and can therefore be derived from a scalar potential. A perfect analogy can be drawn between $\underline{P}$ and the electrostatic field vector $\underline{E}$ in a perfect dieletric.

On the other hand, $\underline{Q}$ and $\underline{J}$ are solutions to the Laplace equation only where the Lagrangian density vanishes. They satisfy

$$\nabla^2\underline{J} = \underline{\nabla}(\underline{\nabla}\cdot\underline{J}) - \underline{\nabla}x(\underline{\nabla}x\underline{J}) = \underline{\nabla}(\underline{\nabla}\cdot\underline{J}) \tag{9}$$
$$\nabla^2\underline{J} = -j2\omega\underline{\nabla}L \ .$$

Thus $\underline{Q}$ must satisfy the vector Poisson equation, with a source distribution, $2\omega\underline{\nabla}L$. The electrostatic analogy for $\underline{Q}$ involves a space charge in proportion to the Lagrangian.

Now consider the gradient of the time-averaged potential energy:[4]

$$\underline{\nabla}V = (1/4\rho c^2)\,\underline{\nabla}(p\cdot p^*)$$
$$= (1/2\rho c^2)\,\mathrm{Re}(p\underline{\nabla}p^*)$$

$$= (\omega/2c^2)\ \mathrm{Re}(+jp\underline{v}^*)$$

$$= -(\omega/c^2)\ \mathrm{Im}(\underline{J}),\quad \text{or}$$

$$\underline{\nabla}V = -\ \omega\underline{Q}/c^2 \quad . \tag{10}$$

The combination of Eqs. (6) and (10) yields

$$\nabla^2 V = 2k^2 L, \qquad k \equiv \omega/c \quad . \tag{11}$$

Thus, V is the solution to the scalar Poisson equation with a source distribution, $-2k^2L$.

The specific acoustic wave admittance vector for the sound field may be defined as

$$\underline{Y} \equiv \underline{v}/p \quad . \tag{12}$$

Let us compute its magnitude, with the aid of Eqs. (1):

$$Y^2 = \underline{Y}\cdot\underline{Y}^* = \underline{v}\cdot\underline{v}^*/p\cdot p^*$$

$$= T/\rho^2 c^2 V$$

or

$$Y^2 = \frac{1}{\rho^2 c^2}(1-\frac{L}{V}) \quad . \tag{13}$$

The magnitude of the admittance is $(\rho c)^{-1}$ wherever L vanishes.

It is instructive to integrate Eq. (5) over a volume $\mathcal{V}$ closed by a surface S. By the application of Gauss' Theorem, one obtains

$$j2\omega \int_F L d\tau = - \int_F \underline{\nabla} \cdot \underline{J}\, d\tau = - \oint_S J_n\, d\sigma$$

$$= - \frac{1}{2} \oint_S p v_n^* \, d\sigma \quad , \tag{14}$$

where the subscript n indicates the normal, outward-pointing component of the vector to which it is appended. Consider a region F which is an infinite fluid medium surrounding a single sound source, and let the surface S be chosen as the combination of the surface of the sound source and the "sphere at infinity." Far from a sound source and far from any boundaries, the pressure and particle velocity vary as $(1/r)$, r being distance from the source, and the phase angle between them varies as $\tan^{-1}(1/kr)$.[5/] Thus the imaginary part of that part of the surface integral in Eq. (14) which is taken over the "sphere at infinity" (whose area is proportional to r^2) can be made as small as desired by choosing a large r. The real part of the surface integral yields, of course, the power radiated through the distant sphere.

The remaining part of the surface integral, the contribution from the surface S_o of the sound source, can be written

$$\frac{1}{2} \int_{S_o} p(-v_n)^* d\sigma \equiv \frac{1}{2} Z_o\, v_o^2 \tag{15}$$

where v_o is a real scalar reference velocity magnitude which characterizes the strength of the source. This equation defines Z_o, which can be called the complex surface impedance for the surface S_o. (Note that the normal velocity $-v_n$ is positive when directed out of the source.)

Take as a reference condition $v_o=1$. Then $\frac{1}{2}\,\mathrm{Re}(Z_o)$ is the power radiated to a distance and, from Eq. (14) and the discussion above,

$$\mathrm{Im}(Z_o) = 4\omega \int_F L_{v_o=1}\, d\tau \quad . \tag{16}$$

The integral in Eq. (16) is the net excess of kinetic over potential energy in the fluid surrounding S_o for a unit peak amplitude of reference velocity, v_o.

One can carry out a parallel analysis in terms of admittances. If p_o is a real scalar reference pressure magnitude characterizing the strength of the source, then define a complex surface admittance Y_o by

$$\frac{1}{2} \int_{S_o} p(-v_n)^* d\sigma = \frac{1}{2}\, Y_o\, p_o^2 \tag{17}$$

Then $\frac{1}{2}\, p_o^2 \mathrm{Re}(Y_o)$ is the power radiated to a distance and

$$\mathrm{Im}(Y_o) = 4\omega \int_F L_{p_o=1}\, d\tau \quad . \tag{18}$$

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1. See, particularly, P. J. Westervelt, J. Acoust. Soc. Am. 23, 345-348 (1951); ibid, 29, 199-203 (1957); ibid, 29, 934-935 (1957).

2. For any two harmonic variables a and b, expressed in the present complex notation,

$$\langle \mathrm{Re}(a)\,\mathrm{Re}(b)\rangle_{av} = \mathrm{Re}(\tfrac{1}{2}ab^*).$$

3. P. M. Morse, Vibration and Sound, second edition, (McGraw-Hill Book Company, New York, 1948), chapter VII, section 25.

4. One phase of analyses of transmission irregularities in room acoustics focuses on the maxima and minima, in space, of rms pressure. But the extrema of pressure are coincident with extrema of potential energy, that is with points where $\nabla V = 0$. According to Eq. (10), the extrema of rms pressure are therefore coincident with the points where the Poynting vector $\underline{J}$ is real ($\underline{Q}=0$).

5. See, inter alia, L. E. Kinsler and A. R. Frey, "Fundamentals of Acoustics," (John Wiley and Sons, New York, 1950), chapter 7.

APPENDIX II

INTENSITY MEASURING DEVICES

Introduction

An acoustical intensity meter has the potential ability to trace real sound energy flow in the presence of a strong field of reactive energy; this is its major attraction. To the extent that it can be done successfully, there are several immediately useful applications in the field of underwater sound. Indeed, it may be that such a device can deal with important problems which are not solvable in any other practical way. Some of these applications are as follows:

1) the measurement in the near-field of total power flow to the far-field, either in a free field or in a reverberant tank;

2) prediction of far-field directivity from near-field measurements of the intensity vector. For some sources calculation may be required because the static fields of the various orders of n-poles fall off with distance as different powers of r; an advantage here over Horton's method is that no "time-phase" measurements are required, but only time-averaged data;

3) measurement of the pressure gradient term required by Horton for his "simple Helmholtz solution" (cf. JASA 33, 877 (1961), eqn. 3);

4) measurement of the acoustical impedance of materials for tank linings;

5) measurement of self and mutual radiation impedance of transducers;

6) tracing of energy flow out of tanks, either in the form of unintentional "leaks" or as normal transmission of energy through the tank walls (or the water surface) into the external earth or air; in particular, the intensity meter could quickly answer certain urgent questions about the "natural absorption" of tanks and the diffuseness of the sound field;

7) study of reverberant fields in tanks further to learn what kinds of measurements on transducers are appropriate there.

It is the purpose of this appendix to confront the requirements imposed by these applications with the limitations of the present state of the art.

Principle of Operation

The term "intensity measuring device" is used here to mean any device which measures (i.e., yields a pointer indication of) a true vector quantity, namely the time average of the product of instantaneous local pressure and particle velocity in a sound field in a fluid medium.* In the instruments which have actually been built,

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*In 1960 the American Standards Association established the definition: "Sound Intensity" (Sound-Energy Flux Density) (Sound-Power Density). The sound intensity in a specified direction at a point is the average rate of sound energy transmitted in the specified direction through a unit area normal to this direction at the point considered. The commonly used unit is the erg per second per square centimeter, but sound intensity may also be expressed in watts per square centimeter.

Note 1. The sound intensity in any specified direction, a, of a sound field is the sound-energy flux through a unit area normal to that direction. This is given by the expression

$$I_a = \frac{1}{T}\int_0^T p v_a \, dt$$

where T = an integral number of periods or a time long compared to a period

p = the instantaneous sound pressure

v_a = the component of the instantaneous particle velocity in the direction a.

Note 2. In the case of a free plane wave or spherical wave having the effective sound pressure, p, the velocity of propagation, c, in a medium of density, ρ, the intensity *in the direction of propagation* is given by $I = p^2/\rho c$. S1.1--1960 *American Standard Acoustical Terminology*, New York (May 25, 1960). (Italics are added to emphasize the fact that, since intensity is a vector quantity one may properly .... in fact, one must .... speak of components of intensity in various directions; the maximum "component" will, of course, be that in the direction of net energy flow).

the magnitude is generally read from a meter; the direction of the vector is determined from the orientation of the sensing element. Ideally such a device will respond only to net energy flow, ignoring all reactive components of the field. The meter reading will vary with the orientation of the sensing element in the sound field, giving a maximum positive indication when the sensing element is pointed in the direction of net energy transfer, a negative indication when pointed in the opposite direction, and zero indication when, for example, oriented transversely in a plane progressive wave; in the last case, although there may be considerable energy flow it is in the direction to which the meter quite properly does not respond.

Several such devices have been described in the literature[1-11] but surprisingly little practical use appears to have been made of any of them. These efforts will be summarized later.

In addition, at least two devices have appeared on the commercial market as "intensity meters" which do not correspond to the definition given above. They measure rather the (scalar) energy density and hence are not useful for determining energy flow. Their advantage over previously-used devices for reading squared-pressure and squared-velocity is their extremely small size, which would be suitable, say, for exploring the details of a beam of ultrasonic energy. Their most obvious application to the problem of calibrating large transducers would be as indicators of average energy density in a reverberant field where only the total power output of the transducer is of interest.

There is, however, a mildly interesting possibility which arises from a property that might appear initially as a drawback to these two devices and this might profitably be investigated further. W. J. Fry

of the Biophysical Research Laboratory of the University of Illinois has made a careful study[12] in connection with the design of a similar instrument of his own, and he concludes that it is possible to determine the energy density accurately with his device, but only if pulse techniques are used. There are too many variables for accurate measurement in the steady-state. He appears to feel that this is true for the commercial instruments as well. The significant datum in the pulsed case is the <u>rise time</u> of temperature in the "absorbing medium" when the pulse is received. For our purpose this may lead to an advantage: since only the <u>start</u> of the received pulse is observed, perhaps a more lenient criterion can be used to relate permissible pulse length to tank size. The effect of the Q of the transducer on the pulse shape and hence on the initial phase of the thermal transient would have to be studied in detail, of course.

A further possibility, which might permit even shorter observation times, would be to use a device similar to that of Fry to assess the viscous interaction of the pulsed sound field with the wire of a thermocouple, since this thermal reaction can be observed immediately without waiting for the temperature of a comparatively large absorbing element to change. Such a method would have the further advantage of being sensitive to the direction of travel of the sound.

Since these instruments do not measure true intensity, however, they will not be considered further here.

It has been stated above that "ideally" an intensity meter measures only the net energy flow and ignores reactive power. For example, even if both the pressure and velocity signals at some position are strong, the meter should yield a zero reading when they are

exactly in time-phase quadrature, as would be true in the standing-wave pattern of a non-absorptive space, or if the sensing element is oriented in a null plane for particle velocity in a plane progressive wave. This is an extremely severe test, however, and because of practical difficulties of construction the actual instruments have generally showed non-zero readings under such conditions. In a space which is highly reverberant, therefore, a small net energy flow might be difficult to measure accurately, since it would tend to be obscured by the false response of the instrument to the strong reactive field. This difficulty is likely to plague all practical intensity meters. Indeed, one is puzzled and a bit suspicious over the fact that, although seven instruments have been described in the literature, there are virtually no follow-up reports to indicate successful and useful measurements made with any of them!

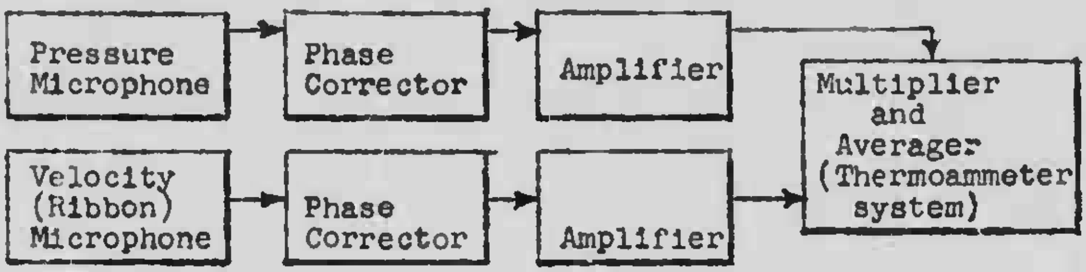
It would, therefore, be worthwhile to assess the limitations of the instruments already described in the literature and subsequently to attempt to estimate what order of precision will be required for the present applications. Then we can determine whether or not in the present state of the art intensity meters can be useful for the study of large transducers in sound fields in restricted spaces.

Previous Efforts and Their Limitations

H. F. Olson

In 1932, a patent was issued to H. F. Olson for "A System Responsive to the Energy Flow in Sound Waves."[1] He had recently invented the ribbon microphone, sensitive to the particle velocity in a sound wave, and this had led to speculation as to how it could be combined with other apparatus in order to measure energy flow, total energy density and the potential and kinetic energy densities

separately. His "Energy Flow" patent application is couched in very general terms and covers all systems based on the simultaneous conversion of "the energy of different components of the sound wave into electrical variations", the subsequent combination of these electrical variations to obtain their product, and the provision of a pointer indication of this product to represent the total energy flow. The disclosure shows that he intends some such arrangement as this:



Clapp and Firestone

An attempt was made by Clapp and Firestone[3] at the University of Michigan in the late thirties to realize an operable intensity meter in precisely the form suggested by Olson. They describe an apparatus which consists of two bimorph crystals placed at either end of a ribbon microphone and so connected as to average their signals, thus approximating the pressure at their geometrical center (which is also the center-position of the velocity-sensitive ribbon). After phase correction and amplification, the two electrical signals (one from the pair of crystals, the other from the ribbon) are fed to a "quarter-square" multiplying device which squares the sum and the difference of the pressure and velocity signals in thermal elements; two thermocouples are arranged to detect the thermal variations in these elements and the thermocouple output signals are subtracted to give the final result. By means of a switching arrangement, they were also able to read out the kinetic and potential energy densities as well as the intensity.

There were several difficulties with this method. The large size of the microphones introduced large diffraction errors at frequencies above 1000 c/s; the phase shift of the pressure transducers was temperature-dependent; the phase characteristics of the ribbon at its many resonances throughout the audio spectrum changed with temperature and transducer orientation and were very difficult to compensate successfully and permanently; because of coupling between the two signal channels, the phase shift was dependent on signal level; and the pressure and velocity were not really measured at the same point in space. The frequency range was restricted to 100-2000 c/s; the dynamic range was not stated.

Nevertheless, the authors, using the intensity meter in an impedance tube, were able to determine the absorption coefficients for red hairfelt and Celotex B in the frequency band between 100 and 2000 c/s; and compared with conventional tube measurements, they found agreement which was "considered good in view of the variability of the material." The advantage of using an intensity meter instead of a pressure microphone in the impedance tube is that the measurement need be made at only one position, as contrasted with standard tube methods which require moving the microphone continuously through the length of the tube. The authors go on to describe a method with which the intensity meter could be used to determine the absorption coefficient under conditions of random incidence and in fact made some measurements of this type but with somewhat less success than in the impedance tube. Also a few measurements of the acoustical "mobility" of the absorptive samples were made. This is the only report in the literature of an actual useful application of an intensity meter!

Enns and Firestone

Shortly afterwards, Enns and Firestone[4], also at Michigan, reported improvements of the original instrument of Clapp and Firestone, which extended the frequency range down to 65 c/s by measuring in narrow bands of frequency and which reduced the extent to which phase shift was dependent on signal level.

Enns and Firestone at about the same time also made some theoretical calculations[13] (which they said were inspired by the new intensity meter) of the outward energy flow from various types of radiating surfaces, but no measurements made with the intensity meter were ever published to accompany the theoretical results.

Bolt and Petrauskas

At MIT at about the same time, Bolt and Petrauskas[5] were exploiting similar ideas to measure the acoustical impedance of an absorptive material. They used two dynamic microphones near the surface of the material, spaced a small distance d apart on the normal to the surface. The sum and differences of the two pressure signals were measured and used to give the normal specific acoustic impedance.

$$\frac{Z}{\rho c} \approx \frac{\pi d}{\lambda} \left| \frac{p_1 + p_2}{p_1 - p_2} \right| e^{i(\theta - \pi/2)}$$

with reasonable accuracy for $\lambda/d > 10$ and for impedances up to about $10\ \rho c$.

Null methods were used both for calibration and alignment and for the taking of data.

The frequency range was 50-500 c/s; the dynamic range of the system is unknown except as implied by the statement above that accuracy was not good for

$$\left(\frac{\pi d}{\lambda}\right)\left|\frac{p_1 + p_2}{p_1 - p_2}\right| > 10 \quad .$$

The equipment was presumably used to measure local impedance, though this is not actually reported; the extent of this use is not known.

Schultz

In the early fifties, a group at Harvard were looking into new approaches to the problems of room acoustics, chiefly employing "parameter-product measurements", such as the various correlation techniques. T. J. Schultz, using newly available plastic materials, had developed small condenser microphones[14] with very smooth amplitude and phase response over a wide frequency range, and these were combined in pairs back-to-back to give the same type of (p_1+p_2) and (p_1-p_2) readings as had been used by Bolt and Petrauskas to measure acoustic impedance.[6-8] In this case, however, (p_1+p_2) was taken as an approximation to the pressure midway between the transducers and (p_1-p_2) as the pressure gradient. Integration of the gradient yielded the particle velocity, which signal was then multiplied by the pressure signal and the resulting product averaged to give a pointer reading of the (vector) sound intensity. The multiplication and averaging were done in a specially-built moving-coil meter.

The major difficulty with the procedure is this: the differencing operation (p_1-p_2) requires for its success an extremely high S/N ratio in the p_1 and p_2 signals, for if p_1 and p_2 are nearly equal,

their difference is small and it is likely to be masked in electronic (thermal/tube) noise and harmonic distortion products. Moreover, this already jeopardized difference signal is the one which must be integrated to give the velocity signal; and the integration operation ($1/j\omega$) by its very nature discriminates against the signal (predominantly high frequency) in favor of the tube noise (mainly at low frequencies).

In principle (i.e., if S/N problems are not severe) this scheme will provide measurements of potential and kinetic energy density as well as intensity; and it can readily be adapted to indicate acoustical impedance by forming the ratio of the (complex) p and v signals to give both amplitude and phase.

But in the present case, because of noise problems, it was necessary both to use high-pass filters to reduce the noise, which was mostly below 90 c/s, and also to perform the integration in the "pressure channel" rather than the "velocity channel". This latter step is permissible since the formula for intensity* is symmetrical in p and v (it doesn't matter which term the $1/j\omega$ is associated with), and it is desirable because it tends to distribute the S/N hazards between the two channels. This, however, precludes the measurement of kinetic energy density and acoustic impedance, since no velocity signal is available in the circuit. If less noisy input circuitry were developed, the integration operation could be restored to the "velocity channel" and energy density and impedance measurements could be made.

The frequency range was 90-10,000 c/s; the dynamic range was 50 db.

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*See footnote on page 92.

A few experiments were made after the intensity meter was completed and calibrated. In a model-scale, rectangular room with thick marble walls, the pressure and intensity were measured as a function of distance along the major axis of the room. The room was excited at a single frequency by an electrostatic loudspeaker whose diaphragm was planar and comparable in area with the cross-section of the room. All modes but one could be suppressed by driving the loudspeaker at the desired modal frequency and by orienting it normal to the propagation direction of the desired mode. Under these conditions, it was expected that the pressure would fluctuate greatly in space because of the standing wave but that the intensity reading would be more or less constant at a very low value, representing practically no real energy flow. Instead, although it was found that the pressure variations were as great as anticipated, there were also moderate fluctuations in the intensity, with peaks occurring slightly farther away from the source than the peaks of pressure. Since this was unexpected and there was not time then to investigate further, the data were not published.

This experiment raises the question now, however, whether this behavior merely demonstrates that a "non-ideal" intensity meter may show false readings when one of the components (here, the pressure) is sufficiently large. On the other hand, since energy must after all be leaving the room somewhere, this may very well be happening near the pressure maxima, in which case the intensity meter could have been giving accurate data. It would be very valuable in the present connection to pursue this question further: Schultz's intensity-measuring equipment, which was originally built for ONR, still exists and could be further exploited at the present time.

Bouyoucos

J. V. Bouyoucos, also at Harvard, proposed a modification of the same equipment using hydrophones to measure intensity in the outlet ducts of a hydraulic oscillator.[15] The pressure transducers in this case were to be annular sections of the duct made of piezo-electric material. He also proposed[16] measuring acoustical impedance with equipment similar to that of Schultz, in which the only measurements required are the ratio of pressure difference ($P_D = |p_1 - p_2|$) to pressure sum ($P_S = |p_1 + p_2|$) and the phase angle between P_D and P_S. (Note the similarity to the method of Bolt and Petrauskas.) Whether or not any measurements were ever made with this modified equipment is not known.

Baker

During the same period, Stuart Baker[9] at MIT made an intensity meter using a crystal microphone for the pressure signal and a biased hot-wire anemometer for the velocity signal. The signals from these transducers were multiplied and averaged by electronic means to yield intensity amplitude. No attempt was made to measure the kinetic or potential energy densities or the acoustical impedance, and in fact, almost no data for acoustical intensity itself were shown in the report; the published results include only some instantaneous phase relationships between the electrical outputs of the pressure and velocity microphones as shown in Lissajous figures, and a curve, made with the probe at a fixed position of maximum intensity in an impedance tube, to show that as the pressure level increased, the intensity level reading also increased proportionally; no other intensity data are given.

The velocity signal was derived from a hot-wire anemometer in a constant-current circuit. In order to transduce the alternating component of acoustical velocity, it was necessary to supply a steady biassing air flow, for without this the hot-wire cannot distinguish between positive and negative flow directions. A small protective tube was placed around the hot-wire, both for protection and to direct the biassing air flow; also, since the unshielded hot-wire would be sensitive to particle velocity in all directions, the addition of the tube tends to give the velocity transducer a directivity approximating the required cosinusoidal pattern. Unfortunately, it did not yield a very good approximation. Moreover, the air flow which provided the necessary bias (of several hundred cm/sec) also introduced troublesome aerodynamic noise, mainly below 70 c/s.

The dynamic range of the instrument was about 35 db, with a frequency range of 60-7000 c/s. When the bias velocity was increased to increase the dynamic range, the aerodynamic noise also increased so that no net improvement could be realized.

Nothing is stated about the actual use of the equipment in measuring intensity.

H. F. Olson

Around 1953, H. F. Olson[17] adapted for intensity measurement a standard unidirectional microphone (RCA Type 77-C1), which contains a ribbon suspended between the poles of a magnet structure. Half the ribbon was exposed to the medium on both sides and responded to velocity; the other half was exposed on one side only and responded to pressure. These sections were connected to separate amplifiers and were multiplied in an electronic wattmeter. Olson says he has used this instrument over the frequency range from 100 to 2000 cycles and implies that the success is about the same order as that of Clapp and Firestone. Nothing has been published about this work.

Awaya, Yokoyama, Shirahata and Ito

In 1959 at the University of Tokyo, Messrs. K. Awaya, I. Yokoyama, A. Shirahata and M. Ito developed a fairly elaborate acoustical wattmeter.[10] The report is in Japanese and no translation is available, but it is clear from the mathematical development, as well as from the English captions for the curves and photographs and from the circuitry, that this is a well-designed realization of Schultz's method. Two condenser microphones are used in the same "wafer-configuration" to measure $(p_1+p_2)\approx p$ and $(p_1-p_2)\approx\nabla p$. The ∇p signal is integrated and the resulting velocity signal is multiplied with the pressure signal and the product averaged electronically to give an intensity measurement. Provisions are also made for measuring p^2.

The frequency range is 200-5000 c/s; the dynamic range is unknown.

Nothing can be gleaned from the untranslated article concerning actual applications of the instrument.

Boyer

The latest intensity-measuring device described in the literature is that of G. Boyer at the David Taylor Model Basin.[11] This instrument was developed specifically for underwater use and, in fact, was intended to apply to the problem of calibration of large transducers.

The pressure signal is supplied by a conventional cylindrical hydrophone. The velocity signal is derived from an accelerometer output signal which is, of course, proportional to the pressure gradient. This signal is integrated to give velocity and, once more, this and the pressure signal are multiplied and averaged in an approximate fashion in a polarity coincidence correlator.

There was great difficulty in calibrating the instrument because of the lack of a convenient underwater free field; in fact, no calibration technique has yet been worked out for the instrument. As well as the author could tell from his measurements and the subsequently applied corrections, the phase shift between channels was less than $\pm 7^{o}$ and the amplitude response was substantially uniform from 150 - 3000 c/s; but he feels that the useful frequency range is 50 - 5000 c/s.

Nothing is stated about any application of the instrument.

Present Requirements vs the State of the Art

In all the cases reported in the literature, the problems of phase shift and noise in the instrumentation appear to be serious; and underwater calibration of an intensity meter will, because of the absence of a convenient free field, be difficult from the operational standpoint. No doubt some improvement can be made in these areas, but certain limitations will surely persist. It would, therefore, be useful now to try to determine the requirements for the proposed applications, to see if they are consistent with the present state of the art in the use of the intensity meter. Unfortunately, to determine these requirements turns out to be rather difficult.

Moreover, it has been surprisingly difficult even to assess the present state of the art in a form useful for our purposes. There is a discouraging sameness in all the published reports: a typical one begins by stating the useful applications which an intensity meter will find, then describes the classical difficulties of excessive phase shift (mostly in the transducers) and equipment noise of one kind or another which limit the frequency and dynamic ranges of the instrument; some type of calibration procedure is given which is usually not very complete; either no actual application at all is described or at most only

a very limited one; the author usually recommends further work to clear up the limitations of the "present model;" and then nothing more is heard of it! This lack of actual useful application of the intensity meters is puzzling and a bit disturbing. It raises a broad question: could it be true in general that the subject of energy transfer in room acoustics is so complicated that its phenomena often elude our intuition? Some of the difficult questions raised elsewhere in this report (and the single example of Schultz's unexplained experimental data) make the suggestion at least plausible! Perhaps others of the reported intensity measuring devices, behaving exactly as they should, have yielded correct data which seemed inexplicable at the time (and therefore unpublishable) only because they were unexpected; this might account for the scarcity of reported applications of these instruments. More work is sorely needed on this very fundamental point, for an uncertainty of this kind makes it almost impossible to assess the present limitations of the intensity meter.

Energy Flow in a Tank

Acknowledging from the start that quantitative results will be few and uncertain, let us begin by reviewing some of the conditions under which intensity measurements might be made. Consider first the case of a non-directional sound source in a semi-reverberant tank whose walls are irregularly covered with patches of sound absorbent material or with sound energy "leaks". Outside the region where the direct field* of the source predominates over the reverberant field** in the tank nothing can be learned about the performance of the sound source except its total power output, for the distribution of energy flow here will be governed almost wholly by the distribution of absorption and not by the directivity of the source. Referring to Figure 1,

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*Defined as the sound field that would exist about the source in free space.

**Defined as the difference between the direct field and the field which actually exists in the tank.

within the dotted circle the direct field predominates and the outward energy flow is uniformly distributed in angle about the (non-directional) source. Outside this circle, however, the net energy flow must arrange itself so that it streams toward the absorbent patches, since it cannot leave the tank elsewhere. An intensity meter moved about on the dashed contour, for example, would discover "lobes" at A, B, and C where the energy flow is more dense than at intermediate positions. These lobes, of course, have nothing to do with the directional properties of the source, though they will have an interest for someone who is studying the tank itself: indeed, if one is looking for energy flow out of a tank, only an intensity measurement is of much use.

Now, however, suppose that in the same tank the same total amount of absorption is uniformly distributed, as in Fig. 2 (ignoring for a moment the complication of the free upper surface). Even with a uniform distribution of absorption, we find that interference phenomena at the boundaries lead to a rather complicated build-up of sound-pressure which is such as to cause more energy to be absorbed in the corners of the tank than at the center of the wall panels. Thus, the energy flow tends to concentrate at the corners, much as it did near the absorbent patches in the previous case. This abnormal pressure distribution extends out into the tank for a distance of about half a wavelength.

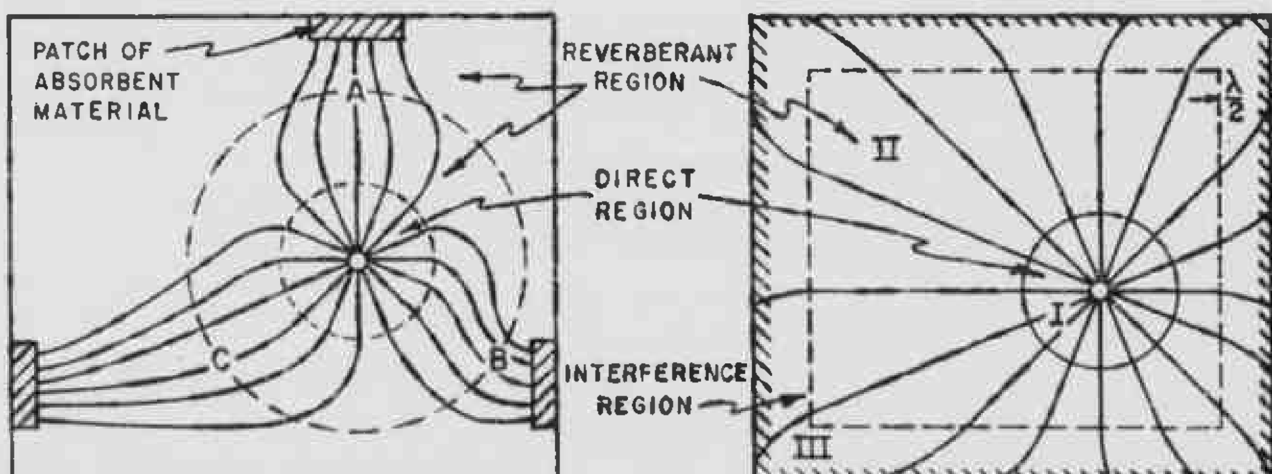

FIG. 1 FIG. 2

Three Regions of Interest in a Semi-reverberant Tank

This discussion can be summarized as follows: In a very large (but not infinite) tank we can define three regions of interest:

1) a region surrounding the source in which the direct field predominates over the reverberant field and in which the directional characteristics of the source are discernible, as in free space;

2) a region outside the direct field where the reverberant field predominates, and where the energy distribution is more or less homogeneous and independent of the detailed characteristics of the source as well as of the tank*;

3) farther away from the source, within $\lambda/2$ of the tank walls, is an interference field in which the pressure rises sharply as the walls are approached.

In this last region, an intensity meter would find dense energy flow into the corner locations; also, because of the pressure rise near the walls, the average pressure is greater here than that taken throughout the total volume of the tank. This region appears most prominently for the case of low average absorption; it tends to vanish as the absorption becomes very large.

*We are still assuming that the wall absorption is more or less uniformly distributed.

Only in a tank which is very large compared to a wavelength will these three regions be distinct (and then possibly only for a small source). If, in addition, the tank absorption is rather high so that the reverberant field is weak, the direct-field region may extend out to include not only all of the near field of the source, but some of the far field as well (see the discussion in Chapter II). This condition would be almost ideal for sound-source measurement, since one could then determine the directivity and total power output with simple pressure measurements in the far field of the source, with no interference from the reverberant field of the tank except, perhaps, at nulls between the lobes of the directivity pattern. It was shown in Chap. III (eqn. 29, ff.) that to achieve this desirable situation (under particular assumptions about the transducer), the volume of the tank must be at least $1.4 \times 10^4 \lambda^3$. At a frequency of 4,000 c/s, for example, this would require a tank with a typical dimension of at least 30 ft. In a tank which is barely large enough to meet this size criterion, almost perfect absorption would be required to include the far-field of the source within the direct field, which in this case would extend practically to the tank walls, the interference field near the walls being virtually eliminated. If only moderate absorption is attainable, however, the tank volume would have to be much greater than 30 ft. The problem would become even more acute at lower frequencies. Evidently, it is not practical to hope to achieve this nearly ideal condition.

Possible Measurements in the Three Regions

Actual tanks, of course, are seldom large and are only moderately absorbent. Let us see, then, what measurements are possible for a practical situation in each of the three regions. In a typical tank, the direct field extends only a short distance from the source before

it is lost in the reverberant field; thus, only the near field of the source is undisturbed and accessible for measurement, and it contains both real and reactive power*. It is in this region (i.e., the direct field--near field) that the intensity meter is expected to distinguish between the real and reactive components and to give an accurate indication of the real power flow from the source to the far field.

In the interference region, as was mentioned above, the energy flow is governed mainly by the tank configuration and the distribution of absorption therein; an intensity meter in this region will not avail to give information about the distribution of energy flow leaving the source. When sound sources are to be measured, then, measurement positions in the region near the tank boundaries must be avoided altogether, unless the average absorption of the tank can be increased to nearly unity. An intensity meter in the interference region would, however, be extremely useful in tracing energy losses through the walls of the tank.

The situation in the intervening reverberant region is curious and interesting and it deserves further study. Since the net (i.e., real) energy flow must be continuous from source to sink, we know that just outside the direct field it must be similar to that of the direct field, conforming to the directivity pattern of the source; and near the interference field it must conform to the absorption configuration of the tank. Between these limits there must be a smooth transition. It must be remembered, however, that the real energy flow, everywhere within the reverberant and interference regions, is very much less than

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*Even in the near field, however, some attributes of the far field may be recognizable for certain sources (see Chap. II).

the predominantly reactive energy in the standing waves characteristic of reverberant fields. With actual transducers, in tanks of practical size, and at frequencies in the mid-audio range, the real power field is always exceeded by a field of reactive power of one kind or another; either in the direct (and near) field, where the reactive component of the radiation impedance dominates the real component; or in the reverberant region where the accumulated energy in the multiple, overlapping standing-wave system obscures the comparatively small net flow of energy from source to sink. What is critical in the latter case is that the standing-wave system is itself not purely reactive, for a certain amount of the reverberant energy must be absorbed at each encounter with a boundary. The real component of energy flow in the reverberant field in most cases is of the same order of magnitude as the energy flow from source to sink, and when this is true an intensity meter in the reverberant region will not accurately measure the desired local energy flux from the source for it must also respond to the real component of the reverberant field. The only useful information about the source which can be gained in the reverberant region in most cases is a measure of its total power output.

We conclude, then, that from the very nature of the sound field in semi-reverberant tanks all detailed measurements of the source must be made in the direct field (or very near it), and under these conditions the intensity meter can help by separating the real and imaginary parts of the power, since it is the real part of this power which is radiated to the far field and which we are looking for.* Here, again, one must be cautious, for if the direct field happens to be highly

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*Once this point is appreciated, the acoustic intensity meter is seen to have an advantage for near-field measurements even in open water, for example, if the far field is too distant ($r \gg 12\lambda$) to be conveniently reached for instrument placement, or if S/N problems of the open sea begin to intrude at distances beyond the near field.

reactive, the mere fact that it dominates the reverberant field does not guarantee that the real part of the reverberant field will be negligible compared to the real part of the direct field. Once more, even an ideal intensity meter may give misleading readings.

Quantitative Evaluation

It has been difficult to formulate satisfactory quantitative criteria for the performance of the intensity meter in a tank, even in the restricted direct-field region. A few, highly approximate calculations do, however, give encouragement for further investigation. The first of these is an estimate of how the real power in the direct field compares in magnitude with the real power of the reverberant field of a typical tank.

If the total power radiated into the tank by the transducer is W watts, the power W_d supplied by the source to the direct field equals the power absorbed upon the first reflection

$$W_d = W\bar{\alpha}$$

where $\bar{\alpha}$ is the average energy absorption coefficient* of the tank walls.[18] Then the power W_r supplied by the source to the reverberant field is

$$W_r = W(1-\bar{\alpha})$$

*$\bar{\alpha} \neq \bar{a}$, the Sabine coefficient, which parameter is unequivocally determined by reverberation measurements; $\bar{\alpha}$ is difficult to define unambiguously[19].

and this must equal the total energy being removed from the reverberant field each second by absorption at the tank walls. We may associate a p_{rr}^2 component with this energy loss and regard this as an approximation to what we have been calling the "real part of the reverberant field" (the subscript rr signifies "real reverberant"). Assuming p_{rr}^2 to be more or less uniform throughout the tank, we can say that near the boundaries energy is leaving the tank at the rate

$$W_r = \int \bar{I} dS = \frac{\overline{p_{rr}^2}}{\rho c} S$$

where $\bar{I}$ is the average intensity; equating this to $W(1-\bar{\alpha})$, we find

$$\overline{p_{rr}^2} = \frac{\rho c W(1-\bar{\alpha})}{S}$$

Now, equation 11 of Chapter IIB gives for the average squared pressure associated with the <u>total</u> reverberant field

$$\overline{p_{tr}^2} = \frac{4\rho c W}{S\bar{a}} \qquad \text{(II B-11)}$$

and, by definition, at the boundary between the direct and reverberant regions this equals the direct field. Thus, anywhere <u>within</u> the direct-field region the direct squared pressure p_d^2 will be greater than p_{tr}^2. Therefore, within the direct region ... but far enough ($r>\lambda/3$) from the source that the direct field is mostly real ... the ratio U of real direct power to real reverberant power is approximately

$$U \equiv \frac{\overline{p_d^2}}{\overline{p_{rr}^2}} \geqslant \frac{\overline{p_{tr}^2}}{\overline{p_{rr}^2}} = \frac{4}{\bar{a}(1-\bar{\alpha})} \quad .$$

The equality holds at the boundary between the direct and reverberant fields; for example, if $\bar{a}$=0.2($\bar{\alpha}$=0.18), the ratio U at the outer limits of the direct field is about 14 db. The dominance of the direct field rapidly increases as the source is approached.

So far we have considered only the space-average values for the squared-pressure terms; no allowances have been made for the fact that fluctuations of $\pm$ 5 db are to be expected in p_{tr}^2; this is quite proper, however, because such variations should not appear in p_{rr}^2.

It appears, then, that even at the outer boundary of the direct region the real component of the direct field sufficiently predominates over the real reverberant field to make source measurement feasible in this region. Moreover, at least for a point source (monopole), when $r>\lambda/6$ the reactive part of the direct field is less than the real part and it is highly unlikely that any of the reported intensity meters would have difficulty in separating these components.

A second approximate evaluation of the utility of an intensity meter is concerned with the near field of a source and departs from Junger's work on the radiation loading of cylindrical and spherical surfaces.[20] In this formulation, the radial fluid particle velocity at a position given by the radial distance r and the polar angle ϕ in the outgoing wave due to a radiating cylinder is

$$u(r,\phi) = \Sigma\, u_n \cos n\phi \quad .$$

The corresponding pressure is equivalent to a summation of pressure components, each of which is associated with a partial wave:

$$p(r,\phi) = \Sigma\, u_n\, Z_n \cos n\phi = \Sigma\, p_n \cos n\phi \quad ,$$

where Z_n are the corresponding complex specific acoustic impedances. A criterion can be formulated in terms of the phase angles of these impedances.

If $ka \gtrsim 5$, $Z_o \approx \rho c$, so $\frac{Z_n}{Z_o} \approx \frac{Z_n}{\rho c} \equiv \theta_n + i\chi_n$. Junger plots θ_n and χ_n vs ka, with the partial-wave index n as parameter. It can be seen from Figs. 1 and 2 of Junger's paper that for any $ka \gtrsim n$, the real part of the impedance exceeds the reactive part, and more so the greater the value of ka; for $ka < n$, however, the reactive component dominates. Thus, the phase angle of any partial wave does not greatly exceed 45^{o} <u>except</u> when the partial wave is "below cutoff", i.e., when $ka < n$, in which ease the partial field is principally reactive. When $ka > n$, a typical intensity meter will certainly respond with entirely acceptable accuracy to a single partial wave. Whether this conclusion is valid for the summation of a <u>series</u> of waves is a question which must be further investigated.

A similar analysis with similar conclusions can be made for the case of spherical radiators, drawing upon data from the same paper of Junger.

A third approximate evaluation may be drawn from unpublished data of Schultz (described above, p.101) under the assumption that the peak reading of intensity obtained at a position near the peak of potential energy density was, in fact, a false reading. This will give an estimate of the maximum error that might be anticipated under extremely severe measuring conditions. In the (5,0,0) mode, for example, the peaks of pressure and intensity were measured as:

$$p_{\substack{max\\rms}} = 23 \text{ dynes/cm}^2.$$

$$I_{max} = 1.8 \text{ ergs/sec/cm}^2.$$

This pressure would correspond to an intensity in a plane progressive wave in a free field of

$$\frac{p^2_{\substack{rms\\max}}}{\rho c} = \frac{(23)^2}{42} = 12.6 \text{ ergs/sec/cm}^2.$$

In the marble box, however, a standing wave was set up; only one (axial) mode was excited and we are assuming that energy was leaving the box only at the end walls normal to the propagation direction. If the absorption coefficient for marble is α=0.02, then the intensity in the axial direction was

$$I_x = \alpha \frac{p^2_{\substack{max\\rms}}}{4\rho c} = 0.02 \frac{(23)^2}{4(42)} = 0.06 \text{ ergs/sec/cm}^2.$$

Under the (uncertain) assumption that the observed reading of 1.8 ergs/sec/cm^2 was a false reading, the intensity meter was indicating a value 15 db too high. In extreme conditions of dominance of a reactive field over the real field, the greatest errors are to be expected. In this experiment, the real field was virtually non-existent and the error (if it was an error) was, of course, much greater than it would be for a typical situation.

Actual transducers, of course, are far from being as simple as the sources we have just discussed, but it has appeared impossible within the scope of the present project to determine more accurately the relationship between the local real and reactive components of power near an actual source.

Conclusion

It appears that the outlook for the use of an intensity meter in measuring the detailed sound output of underwater transducers is promising, to say the least, and should certainly be further explored. Only an adaptation of appropriate underwater transducers is required. In addition, the application of the intensity meter for measuring pressure gradient is valuable and straightforward. Some development will be required for measuring the acoustical impedance of materials for tank linings, but this, too, is feasible, for, as Bouyoucos has shown, one does not actually require a velocity signal and hence the S/N ratio problem should not be too severe. Presumably, the same is true for determining the self- and mutual-impedances of transducers. It can also profitably be used with virtually no development to investigate such questions as where sound energy leaves the tank. All of these applications require the provision of underwater transducers, but the intensity meter as it stands would find immediate, valuable application for a general study of the properties of energy distribution and propagation in reverberant fields, with a view to forming a clearer understanding of the phenomena and a better appreciation of the limitations of measurement under these conditions.

REFERENCES

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